MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXIV. APRIL 11, 1924. No. 6

DR. J. L. E. DREYER, PRESIDENT, in the Chair.

John Merrill Poor, A.B., Ph.D., Professor of Astronomy, Dartmouth College, Hanover, N.H., U.S.A.;
Bancroft Walker Sitterly, A.M., Ph.D., Assistant Professor of Astronomy, Van Vleck Observatory, Middleton, Conn., U.S.A.;
James Stokley, jun., B.S., Central High School, Philadelphia, and 4531 Osage Avenue, Philadelphia, Pa., U.S.A.; and
Rev. Bernard Guy Swindells, S.J., B.Sc., A.R.C.Sc., Stonyhurst College, Blackburn,

were balloted for and duly elected Fellows of the Society.

Sixty presents were announced as having been received since the last meeting.

On the Influence of Comparison Stars on Photographic Proper Motions. By J. Jackson, M.A., D.Sc.

§ 1. The proper motions of spiral nebulæ determined by Van Maanen are so large that some astronomers have doubted their reality and hesitate to accept them. The actual quantities measured on the photographs are extremely small, for even with the maximum interval of fifteen years a proper motion of 2″ a century gives a displacement of only 0″·3. When we consider that most of the photographs were taken at the 25-foot focus of the 60-inch reflector, that the exposures reached several hours, sometimes spread over two or even three nights, and in any case extending large distances from the meridian, it seems astounding that such accuracy should have been obtained in the measurement of nebular condensations. The measures, however, are extremely consistent,

indicating probable errors much smaller than the quantities measured, and the results obtained from the seven nebulæ investigated agree so well as practically to disarm suspicion. Dr. Smart (*M.N.*, **84**, 333) has just discussed the effect of non-symmetrical distribution of the nebular points, but found that the results could not be seriously altered. In this note it is proposed to discuss the influence of the *random* motion of the comparison stars.

When proper motions are determined from the comparison of photographs taken at two epochs the measured displacements ξ, η have to be reduced to the actual displacements μ_α, μ_δ by equations of the form

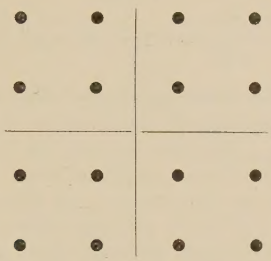
$$\mu_\alpha = \xi + a + bx + cy + dx^2 + exy + fy^2 + \ . \ . \ .$$
$$\mu_\delta = \eta + a' + b'x + c'y + d'x^2 + e'xy + f'y^2 + \ . \ . \ .$$

where x, y are the coordinates of the star measured in any convenient units and $a, b, \ . \ . \ . \ a', b' \ . \ . \ .$ are the "plate constants." To determine these constants a number of stars are selected, for which it is assumed that μ_α, μ_δ, or at least their variations from definite constants, can be neglected. It is well known that in general the mean values of μ_α, μ_δ, will not be zero, and hence the deduced proper motions are called relative proper motions. Usually the faintest well-defined stars on the plate are used for the determination of the plate constants, so that the reduction from relative to absolute proper motion should not exceed a very few tenths of a second of arc per century. The determination of this correction is, however, a problem of considerable difficulty and uncertainty. It is generally assumed that the correction must be directed towards the solar antapex determined from the brightest stars, although there is some evidence that the apex of motion for faint stars may differ considerably from this. If we use the mean of the proper motions of the faintest stars for the determination of the constants of reduction we must derive it from an uncertain extrapolation of the known systematic motion of the brighter stars. If we use only the brighter stars the number available may be rather limited, and the inclusion or exclusion of one may make a considerable difference. There is also the possibility of a magnitude correction affecting the result. Theoretically the correct way is to determine the plate constants from stars for which μ_α, μ_δ are known, but it is improbable that at the present time this method could be successfully applied to any area of the sky, as the probable errors of most proper motions are at least of the same order as the total proper motions of the faintest stars. The reduction from relative to absolute proper motion may however in some areas be made by means of the known proper motions, although here again a magnitude error may be introduced.

The purpose of this paper is, however, not to discuss the reduction from relative to absolute proper motion, but to consider the influence of the random proper motions of the comparison stars on the plate constants. In his reductions Van Maanen included squared terms. It is clear that if only 6 stars are used, and if $a, b, \ . \ . \ . \ f'$ are treated as quite independent, the plate constants will be determined to make the proper motions of all the comparison stars vanish. We proceed to investigate

the influence of the comparison stars in certain ideal cases for which the number of stars is not large.

§ 2. Consider the case in which there are 16 comparison stars placed at the corners of a lattice of squares. The sixteen points are obtained



by giving x, y the values $-3, -1, +1, +3$ in all possible combinations. Writing ζ for $\mu_\alpha - \xi$ we find the normal equations for the determination of $a, \ldots f$ as follows:—

$$\begin{aligned}
16a + \quad . \quad &+ 80d + \quad . \quad . \quad + \quad 80f = \Sigma\zeta \\
. \quad . \quad 80b \quad &. \quad . \quad . \quad . \quad . \quad . \quad . \quad = \Sigma x\zeta \\
. \quad . \quad . \quad 80c \quad &. \quad . \quad . \quad . \quad . \quad . \quad = \Sigma y\zeta \\
80a \quad . \quad . \quad &+ 656d \quad . \quad . \quad + 400f = \Sigma x^2\zeta \\
. \quad . \quad . \quad . \quad . \quad . \quad . \quad &400e \quad . \quad . \quad = \Sigma xy\zeta \\
80a \quad . \quad . \quad &+ 400d \quad . \quad . \quad + 656f = \Sigma y^2\zeta.
\end{aligned}$$

If ϵ be the probable error of ζ we deduce the probable errors of $a, \ldots f$ as follows:—

$$\epsilon_a = \sqrt{\frac{33}{128}}\epsilon = 0{\cdot}51\epsilon$$

$$\epsilon_b = \epsilon_c = \sqrt{\frac{1}{80}}\epsilon = 0{\cdot}11\epsilon$$

$$\epsilon_d = \epsilon_f = \sqrt{\frac{1}{256}}\epsilon = 0{\cdot}06\epsilon$$

$$\epsilon_e = \sqrt{\frac{1}{400}}\epsilon = 0{\cdot}05\epsilon.$$

If the squared terms had been neglected, or if values could have been assumed for d, e, f, we would have found $\epsilon_a' = 0{\cdot}25\epsilon$, $\epsilon_b' = \epsilon_c' = \epsilon_b$.

When we use 36 comparison stars arranged similarly to the above, we find

$$\epsilon_a = \sqrt{\frac{478}{4608}}\epsilon = 0{\cdot}32\epsilon$$

$$\epsilon_b = \epsilon_c = \sqrt{\frac{1}{420}}\epsilon = 0{\cdot}049\epsilon$$

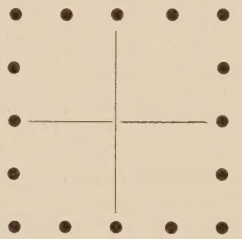
$$\epsilon_d = \epsilon_f = \sqrt{\frac{1}{3584}}\epsilon = 0{\cdot}017\epsilon$$

$$\epsilon_e = \sqrt{\frac{1}{4900}}\epsilon = 0{\cdot}014\epsilon,$$

while $$\epsilon_a' = \frac{1}{\sqrt{36}}\epsilon = 0{\cdot}17\epsilon\,;\ \epsilon_b' = \epsilon_c' = \epsilon_b.$$

It will be seen from these examples that the introduction of the squared terms has doubled the probable error of a, but left the probable errors of b, c unaltered. The fact is that the normal equations do not separate a, d, f well, and the probable error of the deduced proper motions is a maximum at the centre and falls off considerably as we pass to points where x^2, y^2 have average values. In the first example the probable error introduced through the constants at the points $(\pm 2, \pm 2)$ is $\sqrt{\frac{517}{3200}}\epsilon$ or $0{\cdot}40\epsilon$, at $(\pm\sqrt{5}, \pm\sqrt{5})$ only $0{\cdot}36\epsilon$.

§ 3. Consider now 16 comparison stars arranged at equal intervals along the sides of a square, so that $x = -2, -1, 0, +1, +2$ for $y = \pm 2$,



and $y = -1, 0, +1$ for $x = \pm 2$. The normal equations for $a, b, \ldots f$ are:

$$\begin{array}{llllllll}
16a & . & . & +44d & + & . & + & 44f = \Sigma\zeta \\
. & . & 44b & . & . & . & . & = \Sigma x\zeta \\
. & . & . & 44c & . & . & . & = \Sigma y\zeta \\
44a & . & . & +164d & . & + & 96f & = \Sigma x^2\zeta \\
. & . & . & . & 96e & . & . & = \Sigma xy\zeta \\
44a & . & . & + 96d & . & +164f & & = \Sigma y^2\zeta.
\end{array}$$

And we deduce

$$\epsilon_a = \sqrt{\frac{65}{72}}\ \epsilon = 0{\cdot}95\epsilon$$

$$\epsilon_b = \epsilon_c = \sqrt{\frac{1}{44}}\ \epsilon = 0{\cdot}15\epsilon$$

$$\epsilon_d = \epsilon_f = \sqrt{\frac{43}{1224}}\epsilon = 0{\cdot}19\epsilon$$

$$\epsilon_e = \sqrt{\frac{1}{96}}\ \epsilon = 0{\cdot}10\epsilon.$$

When a similar arrangement is extended to 24 points we find

$$\epsilon_a = \sqrt{\frac{943}{1316}} \quad \epsilon = 0{\cdot}85\epsilon$$

$$\epsilon_b = \epsilon_c = \sqrt{\frac{1}{146}} \quad \epsilon = 0{\cdot}08\epsilon$$

$$\epsilon_d = \Sigma_f = \sqrt{\frac{1883}{340844}}\epsilon = 0{\cdot}07\epsilon$$

$$\epsilon_e = \sqrt{\frac{1}{684}} \quad \epsilon = 0{\cdot}04\epsilon.$$

§ 4. These examples are sufficient to prove that the constant a is very seriously influenced by the introduction of squared terms. It will be seen that the trouble is more serious in the distribution considered in § 3 than in § 2. This is due to the fact that the points lie more nearly on a circle. When the points lie on a circle and are symmetrical about both axes, the normal equations are in fact not independent. For if we take n points on a circle the normal equations are:

$$na + b\Sigma x + c\Sigma y + d\Sigma x^2 + e\Sigma xy + f\Sigma y^2 = \Sigma\zeta \quad . \quad (1)$$

$$a\Sigma x + b\Sigma x^2 + c\Sigma xy + d\Sigma x^3 + e\Sigma x^2y + f\Sigma xy^2 = \Sigma x\zeta \quad . \quad (2)$$

$$a\Sigma y + b\Sigma xy + c\Sigma y^2 + d\Sigma x^2y + e\Sigma xy^2 + f\Sigma y^3 = \Sigma y\zeta \quad . \quad (3)$$

$$a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^2y + d\Sigma x^4 + e\Sigma x^3y + f\Sigma x^2y^2 = \Sigma x^2\zeta \quad . \quad (4)$$

$$a\Sigma xy + b\Sigma x^2y + c\Sigma xy^2 + d\Sigma x^3y + e\Sigma x^2y^2 + f\Sigma xy^3 = \Sigma xy\zeta \quad . \quad (5)$$

$$a\Sigma y^2 + b\Sigma xy^2 + c\Sigma xy^3 + d\Sigma x^2y^2 + e\Sigma xy^3 + f\Sigma y^4 = \Sigma y^2\zeta \quad . \quad (6)$$

and we see that so far as the coefficients of a, d, f are concerned $(4)+(6)$ is r^2 times (1), r being the radius of the circle. The equations are therefore soluble only so far as the coefficients of b, c, e do not follow this law. When the points are symmetrically distributed about both axes these coefficients vanish and the values of a, d, f become indeterminate. It is therefore clearly advisable to use comparison stars at very different distances from the centre if squared terms are used in the reduction.

It is to be noted, on the other hand, that if the distribution is approximately symmetrical the accuracy with which b, c can be determined is not materially influenced by the inclusion of the squared terms —a result of considerable importance in the discussion of Van Maanen's results for the internal motions.

The number of independent constants used in the reductions should be as small as possible. If the optical system were perfect and the scale of the photographs the same for the two epochs only three independent constants should be required, two for centering and one for orienting. Except for quantities which should be calculable we should have to determine from the measures themselves only a, a' and $c = b'$. The quantities b and c' determine the scale in the two directions. The differences between c and b' and between b and c' are due to difference

in the differential refraction and aberration at the two epochs and slight change in the form of the images. With a reflector change in focal length and figure, with a refractor differences in the adjustment of the object-glass, are probably the principal causes in producing differences in the constants which should geometrically be equal. The same causes, together with curvature of the field, may make the squared terms necessary, but in such small fields as those considered by Van Maanen, refraction and aberration can produce but small effect. Differences in the definition, the guiding, in the length of the exposures, and in the development will also produce effects on the form of the images on the individual plates, and so on the constants of reduction. It would be instructive if those who make refined astronomical measurements would show how closely the different coefficients agree with theory.

§ 5. Let us now apply these results to the motions of the spiral nebulæ as determined by Van Maanen. It is first of all necessary to get some idea of the value of ϵ, the probable proper motion of a comparison star combined with the probable error of measurement. There are really no satisfactory data for determining this, for the comparison stars used by Van Maanen are on the average as faint as the 16th magnitude. By a rough graphical extrapolation of the logarithms of the proper motion given in Table XXI. of *Groningen Publication*, 30, we find the following mean value of the centennial proper motion :—

Galactic Latitude	0°–20°	0″·4
,, ,,	20–40	0·9
,, ,,	40–90	1·3.

To this there should be added the error of measurement. This will probably not be less than $\pm 0''\cdot5$ per century. Only two of the seven spirals discussed by Van Maanen are within 40° of the galaxy, so that this indicates that ϵ may be taken as $1''\cdot3$.

Another test of the probable random proper motions may be found from Van Maanen's measures. The following table gives the name of the nebula, the galactic latitude, the number of comparison stars used, the mean residual proper motion, and the largest residual proper motions in the two co-ordinates :—

				Mean.		Extreme.	
				μ_α.	μ_δ.	μ_α.	μ_δ.
		°		″	″	″	″
M.	101	59	32	1·3	1·1	3·9	3·6
M.	51	68	20	1·3	0·8	3·6	3·0*
M.	81	41	14	0·7	0·4	2·1	1·4
N.G.C.	2403	29	24	0·6	0·9	4·0	2·7
M.	94	77	14	0·7	0·8	1·8	2·6†
M.	63	73	21	0·4	0·7	1·3	2·6
M.	33	−31	24	0·4	0·5	1·3	1·5

* Two stars with proper motion, $-14''\cdot0$, $+3''\cdot8$; $-15''\cdot9$, $-3''\cdot4$ rejected.
† One star with proper motion, $+5''\cdot9$, $-11''\cdot8$ rejected.

With regard to the mean proper motions it will be noted that the plate constants were adopted so as to make these residual proper motions as small as possible. The actual proper motions must be larger and the increase should be greatest where the number of stars is least. We probably do not exaggerate the value of ϵ if we take it at 1″ per century in each co-ordinate.

§ 6. Van Maanen has for the greater part employed comparison stars projected outside the nebulæ, but he has also employed a number of stars projected on the nebulæ. Without these the value of a determined would have been subject to a much larger probable error. When we use 40 comparison stars distributed along the sides of two squares as considered in § 3, the inner square having 16 and the outer 24 stars, we find the following :—

$$\epsilon_a = \sqrt{\frac{2146}{13640}}\epsilon = 0{\cdot}40\epsilon$$

$$\epsilon_b = \epsilon_c = \sqrt{\frac{1}{190}}\quad \epsilon = 0{\cdot}07\epsilon.$$

Even with 40 stars a is determined with a large probable error.

Van Maanen has not published the values of his constants or of their probable errors. He has, however, given the co-ordinates of the comparison stars from which it is a straightforward calculation to find the probable errors. We have computed these for the two cases in which only 14 stars were used. The worse case is presented by M. 94. The following determinant indicates the coefficients of a, b, c, $10d$, $10e$, $10f$ in the normal equations :—

+14·0	+ 9·1	− 0·9	+ 26·7	+ 2·4	+17·1
+ 9·1	+266·7	+ 23·7	+ 51·2	+ 2·0	+ 7·7
− 0·9	+ 23·7	+171·0	+ 2·0	+ 7·7	+ 4·2
+26·7	+ 51·2	+ 2·0	+103·9	+13·2	+23·4
+ 2·4	+ 2·0	+ 7·7	+ 13·2	+23·4	− 3·9
+17·1	+ 7·7	+ 4·2	+ 23·4	− 3·9	+34·1

After elimination of e, f we find

+ 4·6	+ 4·7	− 4·6	+11·9
+ 4·7	+264·6	+ 21·7	+44·0
− 4·6	+ 21·7	+167·6	− 6·5
+11·9	− 44·0	− 6·5	+76·8

The greater part of the reduction of the leading coefficient took place on the elimination of f; the result after the next two stages (a further large diminution in the value of the leading coefficient takes place on the elimination of d) is—

+2·7	− 1·5
−1·5	+235·7

From the figures given we easily deduce for this nebula

$$\epsilon_a = 0\cdot61\epsilon$$
$$\epsilon_b = 0\cdot07\epsilon$$
$$\epsilon_c = 0\cdot08\epsilon.$$

For Messier 81, the constants are rather better separated, and we find

$$\epsilon_a = 0\cdot46\epsilon$$
$$\epsilon_b = 0\cdot07\epsilon$$
$$\epsilon_c = 0\cdot06\epsilon.$$

For the other nebulæ for which a larger number of stars were used, it may be presumed that the probable errors are smaller.

If the proper motions obtained by Van Maanen for the spiral nebulæ are treated as entirely due to accidental errors, they indicate a probable error of about $0''\cdot6$ a century in each co-ordinate. If we take the mean of the above reduction we find a probable error of $0\cdot54\epsilon$ for 14 comparison stars, or, say, $0\cdot43\epsilon$ for 21 comparison stars. If, then, we take $\epsilon = 1''\cdot4$ a century we can explain the proper motions of the nebulæ in space. We have previously estimated ϵ as $1''$, although with some uncertainty. The fact that 3 stars out of 150 comparison stars had to be rejected because they showed total proper motions of $16''\cdot3$, $14''\cdot5$, $13''\cdot2$ indicate that ϵ may considerably exceed $1''$.

When we consider the internal motions we find the influence of the comparison stars much less marked. For M. 81 the mean distance from the centre to the nebular points is $5'\cdot8$, and for M. 94 it is only about $1'\cdot3$. This gives for these two nebulæ, at the mean distance of the nebular points, probable errors of $0\cdot38\epsilon$ and $0\cdot10\epsilon$ in the rotational and radial motions. The very small value for M. 94 is due to the fact that the nebula is small compared with the area covered by the comparison stars. For all the nebulæ measured by Van Maanen, except M. 101, the relative distance to which the comparison stars extend is greater than for M. 81, so that the average probable error in the internal motions cannot exceed $0\cdot3\epsilon$. The actual motions found by Van Maanen are of the order of $3''\cdot0$ for rotational and $1''\cdot0$ for radial motion. In one or two cases it might be possible to explain the internal motions as due to the comparison stars, but with seven nebulæ all showing an outward motion along the spiral arms, the explanation cannot be found in the random motion of the comparison stars.

As an example of the influence of single stars on the rotational or radial motion, consider the scheme of § 3 with 15 of the stars fixed and the 16th star at (0, 2) with a proper motion of $5''$ a century in the direction of the x axis. This would give to the nebula at this point a rotation of $0''\cdot45$ a century, with an equal and opposite motion for the opposite point. This again shows that comparatively large systematic motions of the comparison stars would be required to cause the effects obtained by Van Maanen.

So far as the proper motions observed in spiral nebulæ are due to the comparison stars, the position will not be improved when the interval

between the plates is increased. To prevent the comparison stars having an undue influence on the result it will be necessary to employ a larger number of stars. If these are not available within the useful field of the reflector, the proper motions of the comparison stars may be determined relative to a larger number of stars photographed with a refractor, although a very long exposure will be required.

Summary.

A discussion of the influence of the random motion of the comparison stars on the constants of reduction, indicates that for the spiral nebulæ investigated by Van Maanen, a reasonable proper motion of the comparison stars will explain most of the total proper motions of the nebulæ, but will not explain the more interesting internal motions.

1924 *April* 11.

Recent Observations of some Bright-hydrogen-line Stars.

By William J. S. Lockyer, M.A., Ph.D. (Plate 9.)

In a paper communicated last year to the Society on "The Spectrum of ζ Tauri," * it was shown that this star, of type B3*p*, indicated variability with regard to the hydrogen line H_β. Sometimes this line appeared as a pure absorption line (1896), at another time this absorption line was accompanied by a bright component on its violet side (1915) and again strong bright components appeared on both sides (1923).

The spectra of many B-type stars, apart from the hydrogen lines, are characterised by the great strength and diffuseness of the helium lines and the comparative sharpness and faintness of the ionised metallic lines. In many of these stars other bright lines appear in addition to those of hydrogen and helium.

In the continuation of the study of the spectrum of ζ Tauri it was felt desirable for comparison purposes to secure the spectra of several other bright-hydrogen-line stars on the same scale.

Reference was therefore made to a paper entitled "Class B stars whose spectra contain bright hydrogen lines," by Paul W. Merrill, which appeared in the early part of the year 1913.†

The observations were mostly restricted to the years 1911 and 1912. Seven different spectroscopes were employed, and they were all used in conjunction with the 36-inch Lick refractor.

Merrill's programme consisted in observing (1) nearly all Class B stars north of declination $-40°$ that were known to have bright lines, (2) a few stars adjacent to bright line stars, and (3) various other stars selected for specified reasons, such as: suspicion, from the three prism

* *Monthly Notices, R.A.S.*, **83**, No. 5, p. 326.
† *Lick Observatory Bulletin*, No. 237, pp. 162–179.

plates, of a peculiar H_β; weak and diffuse absorption lines similar to those characteristic of bright-line stars.

R. H. Curtiss * in 1920 published a paper on a quantitative investigation of the line widths of the emission components of certain hydrogen and metallic lines in twelve class *Bp* stellar spectra, but he gives no details about the hydrogen lines except whether they possess a bright component or not. All the bright lines recorded by him have been photographed in the present survey with the exception of those in the stars 11 Camelopardalis and β Monocerotis, the spectra of which have not yet been taken.

In 1916 Curtiss also published † a paper on "Changes in the Spectrum of H. R. 985." In the same volume (p. 36) he contributed another paper on "Changes in the Spectrum of f' Cygni," and in the succeeding volume (p. 19) he summarised all the early history of this star up to the year 1921. Brief references to Curtiss' observations are made in this paper under the headings of the individual stars.

Eleven years have now passed since Merrill's systematic observations were made. It was considered opportune, therefore, as a special investigation, to rephotograph the spectra of as many of those stars as possible in order not only to secure the comparisons mentioned above, but at the same time to record the present appearances of those spectra for comparison with previous observations.

The instrument used for this purpose was the same as that employed in the case of ζ Tauri, namely, the 9-inch prismatic camera of the Norman Lockyer Observatory, which has a prism of 45° angle and a dispersion of 46·0 mm. between H_β and H_δ.

Although this instrumental equipment is not so powerful as that used by Merrill, yet most of the brighter objects could be satisfactorily photographed. It is interesting to note that in many cases the present observations show more detail with regard to the individuality of the hydrogen lines and display this detail to a further extent into the violet than do those of Merrill. This fact may be due to the different kind of spectroscope employed, namely, the prismatic camera or slitless spectroscope.

Merrill himself in 1915 ‡ pointed out the superiority of the prismatic camera over the slit spectroscope in this special kind of investigation. In referring to Miss Maury's work on the apparent changes in the broad outside absorption of the hydrogen lines, he states: "the wide objective prism spectra probably give better evidence of greater weight on this point than do slit spectrograms."

The present paper summarises the results obtained from photographs taken mainly since the beginning of last September. Up to the end of December, Barnet "Stella" plates were used, and after that date Ilford "Iso-Zenith" plates (H and D 700).

Twenty-one individual stars have been dealt with, but this involved only eighteen separate star settings, as the three remaining stars (all in the Pleiades group) were always photographed simultaneously on the same plate as η Tauri. Only two negatives here discussed, namely, those

* *Pub. Astr. Obs. Univ. Michigan*, **3**, p. 1.
† *Ibid.*, **2**, p. 39. ‡ *Ibid.*, **1**, p. 136.

of κ Draconis, are not recent but were taken in 1915: these are the only photographs not taken by the writer.

Excluding these two plates 126 negatives altogether have been taken, involving a total exposure time of 111·6 hours and an average time of exposure for each plate 52·1 minutes. Of this number 47 were restricted to ζ Tauri, which monopolised 34·2 hours of the total exposure time. The average exposure for this star was 43·7 minutes.

In order to condense the information as much as possible for publication it was decided to give first a general summary in short paragraphs of the results obtained for each star, following this with a table showing the details of the observations of each of the individual hydrogen lines in a very abbreviated form.

All references to Merrill's observations, when not stated, apply to the *Lick Observatory Bulletin*, No. 237, above mentioned. Since Merrill briefly summarised in his paper observations of other authorities made previous to his investigation, no useful purpose would be served by repeating them here.

Before proceeding to the individual observations of each star a few general remarks may be made with regard to the general appearance of the hydrogen lines.

In the case of double bright lines in some of the stellar spectra here mentioned it is to be noted that in the case of H_β and other hydrogen lines, a well-defined sharp-edged absorption line is flanked on either side by bright borders, and these are superposed generally on very broad dark bands. Proceeding towards the violet, these bright borders (or bright lines) diminish in intensity until only a sharply defined absorption line is superposed on a broad absorption band: this narrow absorption line in its turn disappears, leaving only the broad absorption band.

If the intensities of the double bright lines are unequal then the fainter of the two will disappear first as the violet is approached, and the narrow dark line will appear to have a bright line on one side only, and both of these will still be superposed on a broad absorption band. This appearance is of common occurrence and is indicated in a subsequent table by the single letters R or V, indicating that the red or violet component remains alone.

Occasionally the central narrow absorption stands out very prominently in the centre of the broad absorption band. It is difficult sometimes to tell whether this is not due to the presence of very faint narrow bright borders on each side of it, although these borders cannot be quite defined as bright lines as they are when they are more conspicuous. Merrill evidently also found the same kind of difficulty in defining these conditions: thus in one of several instances he states, "H_γ is a clear absorption line with well-defined edges, which may be very faintly bright." Another instance is given later under the heading H.D.C. 45542.

The general procedure of arrangement in the following paragraphs relating to the individual stars is as follows:—

First the Henry Draper Catalogue number is given, followed by the name of the star, R. A. and Dec. for 1900, photometric magnitude, and spectrum type, all taken from this catalogue. Then follow the Norman Lockyer Observatory's number of each negative to which special reference

is made, together with the year, date, and Greenwich mean time of exposure. After this a brief general description of the hydrogen lines is given, followed by Merrill's own observations from the *Lick Observatory Bulletin* previously mentioned. In a few instances observations of other observers subsequent to Merrill's observations are added.

The following stars represent some of the more interesting cases where the present research has revealed notable changes in the appearance of the hydrogen lines:—

ϕ Persei.	H.D.C. 20336.	ψ Persei.	17 Tauri.
η Tauri.	27 Tauri.	25 Orionis.	ζ Tauri.
υ Cygni.			

H.D.C. 5394 (γ Cas.) $0^h\ 50^m.7 + 60°\ 11'$ Mag. 2·25 Sp. Bop.

Negative No. 356	1923 Sept. 10	$12^h 30^m$
414	,, Dec. 23	5 48

Three excellent negatives were obtained between 1923 September and December. The hydrogen lines from H_β to H_η all exhibit double bright lines on not very broad absorption. H_β is an exceedingly intense close double, the components of which seem about the same intensity and they are superposed on a narrow absorption band. As the violet is approached the intensities of the bright lines decrease rapidly and their separations increase, while the underlying absorption bands become very much broader. It is most probable that, from H_δ towards the violet, the violet components of the doubles are a little stronger than the red components. The spectrum contains numerous other bright lines.

Of the numerous references to early observations of this star, those of Merrill and Curtiss * should be chiefly mentioned here. [See Plate 9, No. 3.]

H.D.C. 10516 (ϕ Per.) $1^h\ 37^m.4 + 50°\ 11'$ Mag. 4·19 Sp. Bop.

Negative No. 355	1923 Sept. 10	11^h	37^m
362	,, ,, 11	11	35
403	,, Dec. 13	6	20
435	1924 Feb. 14	8	6

Four negatives of this star were obtained, and while the two first are practically identical the later two show a prominent change.

All the hydrogen lines as far as H_η appear double bright on broad absorption bands. The only exception is H_ϵ, where the calcium line on the violet side obliterates the bright violet component.

Further, the double bright lines of H_β and H_γ are slightly displaced towards the red in relation to the underlying broad absorption bands. With regard to the relative intensities of the bright lines, in the two earlier photographs the red components are about twice those of the violet. In the two later negatives the intensities are very nearly equal, the violet being possibly a little brighter than the red component. In the case of H_β both bright lines are very intense.

* *Pub. Astr. Obs. Univ. Michigan*, 2, p. 1, 1915.

It is interesting to remark that on two negatives taken on the night of 1923 November 24, with the 12-inch M'Clean prismatic camera the violet components of H_β, H_γ, and H_δ are all very much stronger than the red components.

According to Merrill's observations of 1912 and 1913, H_α, H_β, H_γ, and H_δ all showed double bright lines, but he does not state whether the two components were of the same or different intensities. The next two hydrogen lines, H_ϵ and H_ζ, were according to him "clear, sharp-edged absorption lines."

It is clear that the hydrogen lines are very variable. [See Plate 9, No. 2.]

H.D.C. 20336 (– Cam.) $3^h\ 11^m{\cdot}2 + 65°\ 17'$ Mag. 4·76 Sp. B3p.

Negative No. 357 1923 Sept. 10 $13^h\ 0^m$

Between 1923 September 1 and 1924 January 30 seven negatives of this star were secured. They all exhibit practically identical spectra, but some show a little more detail than others.

The spectrum shows all the hydrogen lines up to H_ζ as double bright on broad absorption bands. The red component is in all cases considerably stronger than the violet, with the exception of H_ζ where the spectrum is not sufficiently dense to judge.

Merrill's observations in 1912 record H_β and H_γ as "double bright on absorption" and H_δ probably "faint bright on absorption."

R. H. Curtiss * has studied the spectrum of this star in detail. The plates he discussed were grouped in two epochs, namely, 1912 October to December, and 1916 January to April. The first group (1912) records a spectrum exactly similar to that obtained at Sidmouth, only the strengths of the components of H_δ are described as "rather indistinct," and those of H_ϵ and H_ζ as "quite indistinct." In the Sidmouth photographs there is no doubt about the relative intensities of those lines.

In the second group (1916) the relative intensities of the components are reversed, the violet being the stronger.

The spectrum of this star is evidently periodic in nature, as the Sidmouth photographs of 1923 reproduce the same appearance as recorded by Curtiss in 1912.

H.D.C. 22192 (ψ Per.) $3^h\ 29.^m4 + 47°51'$ Mag. 4.26 Sp. B5p.

Negative No. 364	1923 Sept. 11	13^h	40^m
427	1924 Jan. 27	11	6
436	„ Feb. 14	10	7

Five plates of this star were secured between 1923 September 1 and 1924 February 14, and they all showed a similar spectrum, with the exception of the last, No. 436.

All the hydrogen lines up to and including H_δ are double bright on very broad absorption bands, and the components are of equal intensity

* *Pub. Astr. Obs. Univ. Michigan*, **2**, p. 39.

in negatives 364 and 427. At H_ϵ only the red component is visible owing probably to the obliteration of the violet component by the neighbouring calcium line. H_ζ and H_η appear as narrow absorption lines superposed on broad absorption bands. In the case of H_β the bright lines are unsymmetrical on the broad absorption band being displaced towards the red. In the last photograph, No. 436, the violet component of H_β is very much stronger than the red component, and both are symmetrically placed on the superposed dark absorption band. In the cases of H_γ and H_δ the violet components are if anything a little stronger than the red. The spectrum contains numerous other bright double lines, or what most probably are absorption lines with bright borders, the same lines as noted in ϕ Persei.

Merrill's observations of 1911–1913 record H_β as double bright, H_γ less marked, and the "succeeding lines so weak as to be inconspicuous or apparently absent, leaving only a sharply defined absorption line."

In referring to Miss Maury's earlier observations* of H_β, in which she pointed out that the "bright band is suspected of motion, since while in most photographs it lies centrally on the dark band, in a few it lies on one or the other side of the centre," Merrill states that "such changes might be suspected from my plates, but could scarcely be established." †

The Sidmouth photographs show the bright lines of H_β as symmetrical and unsymmetrical on different dates. [See Plate 9, No. 9.]

H.D.C. 23302 (17 Tau.) $3^h\ 39^m{\cdot}0 + 23°\ 48'$ · Mag. 3·81 Sp. B5p.

Negative No. 370	1923 Sept.	16	13^h	33^m
422	1924 Jan.	25	8	56
430	,, Feb.	1	8	44

Twelve photographs of this star were taken between 1923 September 1 and 1924 February 27, the three above being the most satisfactory and No. 430 the best.

Negative No. 370 shows H_β, H_γ, and H_δ as very thin single bright lines on broad absorption bands. These bright lines are distinctly displaced towards the red in all three lines in relation to the superposed broad absorption bands, indicating possibly double bright lines with very weak (and invisible) violet components.

On negative No. 422 the bright lines in H_β, H_γ, and H_δ appear to be very probably double and symmetrical on the broad absorption bands, while no separation can be seen on the bright line H_ϵ.

Negative No. 430 shows H_β, H_γ, and H_ϵ double bright, but this duplicity is doubtful in the case of H_δ. The red component of H_γ is distinctly brighter than the violet, and both seem slightly displaced towards the red in relation to the broad absorption band.

While Merrill in his 1911 and 1912 observations noted that H_α was a fine sharp bright line on wide absorption, he states "H_β strong absorption." The Sidmouth photographs thus indicate a change since 1912.

* *Ann. Astro. Obs. Harvard Coll.*, **28**, pt. i. 104, 1897.

† *Pub. Astro. Obs. Univ. Michigan*, **1**, p. 136.

H.D.C. 23480 (23 Tau.) $3^h\ 40^m{\cdot}4 + 23^\circ\ 39'$ Mag. 4·25. Sp. B5.

Negative No.	370	1923 Sept.	16	$13^h\ 33^m$
	399	,, Dec.	9	9 16

The above are the best of twelve photographs taken between 1923 September 1 and 1924 February 27.

They both exhibit H_β as a double bright line on a broad absorption band, and H_γ as indistinctly double bright on a broad absorption band. While in No. 399 H_δ displays only a broad absorption band, No. 370 shows a narrow absorption line on a broad absorption band.

Merrill's observations in 1912 refer only to the lines H_α and H_β, the latter being "weak double bright on broad absorption."

H.D.C. 23630 (η Tau.) $3^h\ 41^m{\cdot}5 + 23^\circ\ 48'$ Mag 2·96 Sp. B5p.

Negative No.	348	1923 Sept.	7	$12^h\ 57^m$
	370	,, ,,	16	13 33
	399	,, Dec.	9	9 16

Twelve photographs were secured between 1923 September 1 and 1924 February 27. While slight differences are shown between them there seems no reason to believe that they are otherwise than due to variable definition on different nights.

Of the three plates here selected numbers 348 and 399 are identical, showing H_β as a clear double bright line on a broad absorption band, with the components about equal in intensity. H_γ, H_δ, and H_ϵ appear as narrow absorption lines on broad absorption bands. Negative 370 shows H_β as above described, but with the violet component probably stronger than the red: H_γ appears as a single bright line on the broad absorption band but displaced to the red. H_δ indicates a single bright line symmetrically placed on the broad absorption band, while H_ϵ seems to be a double bright line on a broad absorption band. Campbell's observations in 1893–4 * showed that H_α was bright, but he stated that "it is well known that H_β, H_γ, H_δ, etc., in η Tauri are dark."

Adams, from photographs taken in 1903,† states that "none of the plates of Alcyone show evidence of the presence of a bright line at H_β."

The observations of Merrill in 1912 describe H_β as "very broad absorption with very faint bright portion within."

The Sidmouth observations thus show distinct variations from previous observers. [See Plate 9, No. 8.]

H.D.C. 23850 (27 Tau.) $3^h\ 43^m{\cdot}2 + 23^\circ\ 45'$ Mag. 3·80 Sp. B8.

Negative No.	370	1923 Sept.	16	$13^h\ 33^m$
	422	1924 Jan.	25	8 56

Twelve photographs were secured between 1923 September 1 and 1924 February 27. They all indicate practically the same characteristics allowing for differences of definition and exposure.

* *Astrophy. Journ.*, 2, p. 178, 1895. † *Ibid.*, 19, p. 342, 1904.

The two best negatives specified above indicate H_β and H_γ as single bright lines on broad absorption bands. This is also the case with H_δ in negative No. 370, but doubtful in negative No. 422.

Merrill's observations of 1912 did not indicate the presence of any bright lines, the simple statement "H_α and H_β absorption" only being made.

Miss Maury in 1889 recorded H_β and H_γ as bright, and Keeler in 1889 and Harvard College Observatory in 1896 recorded H_β as bright. The Detroit Observatory recorded H_β as strong broad absorption in the years 1912–13, 1915, and 1920–21.

H.D.C. 25940 (*c* Per.) $4^h\ 1^m{\cdot}4 + 47^\circ\ 27'$ Mag. 4·03 Sp. B3p.

Negative No. 365	1923 Sept. 11	$14^h\ 36^m$
409	,, Dec. 15	9 25

Five photographs were secured between 1923 September 1 and 1924 February 27, the only differences between them being accountable to variable definition and exposure. The two specified above were the most satisfactory. Negative 365 shows H_β as a strong single bright line on a broad absorption band; the bright line does not appear to be double. Negative 409 shows the bright line as a close double, the red component seems a little stronger than the violet. Both negatives show H_γ and H_δ as double bright lines on broad absorption bands with the red component the stronger. While negative 365 shows both H_ϵ and H_ζ as similar to H_δ, negative 409 does not clearly indicate the duplicity of the bright lines.

Merrill's observations in 1912 give H_β and H_γ bright on absorption but not separated, and H_δ as double bright on absorption. No statement is made about the relative intensities of the components. [See Plate 9, No. 7.]

H.D.C. 35439 (25 Ori.) $5^h\ 19^m{\cdot}6 + 1^\circ\ 45'$ Mag. 4·73 Sp. B3p.

Negative No. 416	1923 Dec. 28	$9^h\ 50^m$

Only one moderately good negative of this star was obtained. H_β is a fairly close double bright line superposed on a broad absorption band, but well displaced towards the red in relation to it. The violet component is very much stronger than the red component. H_γ is very similar, except that the components are farther apart. H_δ appears as a single bright line superposed on the broad absorption band but displaced towards the red. H_ϵ is a faint bright line on a broad absorption band, but it is difficult to state whether it is exactly symmetrical or not.

Merrill in his 1910 and 1912 observations describes H_β as "violet component of H_β the wider and fainter" and H_γ as "faint bright." The Sidmouth photograph thus shows a distinct change since Merrill's observations.

H.D.C. 37202 (ζ Tau.) $5^h\ 31^m{\cdot}7 + 21°\ 5'$ Mag. 3·0 Sp. B3p.

Negative No. 265	1923 Feb.	4	$6^h\ 55^m$
276	,, Apr.	7	8 34
351	,, Sept.	7	15 14
366	,, ,,	11	15 35
371	,, ,,	16	14 25
372	,, ,,	16	15 19
400	,, Dec.	9	10 32

The spectrum of this star has been previously discussed by me in a previous communication,* the observations dealt with having been made during the months of January and February 1923. Since then 33 more negatives have been taken grouping themselves round the two observational epochs, 1923 March 5 to April 26, and 1923 August 5 to 1924 February 1.

Restricting the remarks to the appearances of the hydrogen lines alone, the lines H_γ, H_δ, etc., exhibit themselves in all cases as sharp narrow absorption lines, and they are not superposed on any broad absorption bands, like most of the other bright-line stars of this spectral type. In three or four cases the photographs have suggested a bright detached component on the red side of H_γ, but this is not certain.

Otherwise the only hydrogen line which appears to change is H_β, and this line also is not superposed on any broad absorption band.

The seven negatives selected above give the following appearances of H_β. In negative 265 H_β is double bright, *i.e.* it has bright components on each side of the narrow absorption line. The violet component is undoubtedly brighter than the red. Negative 276 is also similar in this respect, but while the violet component is broader than the red and not detached from the absorption line, the red component is a very narrow line and detached. In this photograph H_γ seems to have a red detached component also.

On negative 351 the violet and red components of H_β are of equal intensity and lie, as far as can be judged, close up to the narrow absorption line; but in negative 366 they appear clearly detached, while in negatives 371 and 372 they are questionably detached. On negative 400 the violet component of H_β is again brighter and broader than the red component, and H_γ seems to have a detached red component.

Thus of the hydrogen lines H_β seems to be the only line certainly subject to variations and H_γ probably variable.

Merrill's observations do not include this star in his programme. Curtiss reproduces a spectrum of this star † taken on 1914 November 22, and illustrates H_β as a sharp absorption line with a bright violet component. This appearance corresponds to one I previously described ‡ from a negative I secured on 1915 January 9, namely, "H_β thin sharp dark line with strong bright component on the violet side."

In the account of his spectrum Curtiss states "This emission is undoubtedly present at the edges of the hydrogen lines well towards

* *Monthly Notices, R.A.S.*, 83, p. 326.
† *Pub. Astr. Obs. Univ. Michigan*, 3, 1923, Plates B and C.
‡ *Monthly Notices, R.A.S.*, 83, p. 328.

the limit of the Balmer series, though not set off by absorption borders as in many spectra of class B*p*."

None of these peculiarities are now visible in the spectra which have been recently obtained at Sidmouth. [See Plate 9, No. 5.]

H.D.C. 45542 (ν Gem.) $6^h\ 23^m{\cdot}0$ $+20°\ 17'$ Mag. 4·06 Sp. B5.

Negative No. 439 1924 Mar. 10 $9^h\ 3^m$

The above negative, the best of three secured, exhibits H_β as a faint double bright line on a broad absorption band, with the violet component slightly brighter than the red. H_γ is a narrow absorption line on a broad absorption band. The remaining hydrogen lines are broad absorption bands only. The spectrum contains numerous other bright lines.

This star is not included in Merrill's observations of 1911 and 1912, but reference is made to it by him in a later publication.*

From an examination of photographs taken by him in 1915 November and December, he states "a feeble bright H_α is seen . . . apparently superposed on a broad weak absorption line. . . . The hydrogen lines other than H_α are rather wide, diffuse absorption lines, but several show a narrow central core, the effect perhaps of invisible bright components."

H.D.C. 109387 (κ Dra.) $12^h\ 29^m{\cdot}2$ $+70°\ 20'$ Mag. 3·88 Sp. B5p.

Negative No. 46 1915 Apr. 13 $10^h\ 54^m$

50 „ „ 20 13 23

No recent negatives of this star have been taken, but the two above were secured in 1915.

Both these plates show H_β and H_γ as double bright lines on broad absorption bands with the red component stronger than the violet. On negative 50 these double bright lines are seen in H_δ and more hazily in H_ϵ, but on negative 46 they are not visible, and the lines appear as narrow absorption lines on broad absorption bands.

Merrill in his observations of 1911 and 1912 records various results as regards H_β. Thus he describes H_β as "perhaps faint bright on absorption" and "somewhat uncertainly bright on absorption" from his 1911 observations, and H_β as "double bright on absorption" and "bright on broad absorption" in 1912. According to him "H_γ, H_δ, H_ϵ, H_ζ appear as diffuse absorption lines" in 1911, but he only refers to H_γ in his 1912 observations and describes this as "hazily double bright on absorption."

R. H. Baker† in 1920 completed a discussion of the spectrum of this star, using photographs taken at the Detroit Observatory in the years 1912–1916 and 1918–1919. Without differentiating between any of the plates, he describes the outstanding features of the spectrum as "broad emission lines are centrally superposed on broader absorption, while narrow absorption lines appear nearly central upon the emission." In other words, the hydrogen lines are double bright on broad absorption bands.

* *Pub. Astr. Obs. Univ. Michigan*, **2**, p. 181, 1916.

† *Ibid.*, **3**, p. 29, 1923.

H.D.C. 191610 (b^2 Cyg.) $20^h\ 5^m{\cdot}7$ $+36°\ 33'$ Mag. 4·82 Sp. B2p.

Negative No. 382 1923 Oct. 31 $6^h\ 28^m$
384 ,, Nov. 11 7 20

Of the five negatives taken the two selected above show clearly that H_β and H_γ are both double bright on broad absorption bands, with the violet component a little brighter than the red. H_δ in the first negative appears as a narrow absorption on a broad absorption band, but this is not seen in the second negative.

Merrill, from his observations in 1912, gives different accounts of H_β according to the spectrographs he used. His descriptions are: "Violet component of double bright H_β is stronger than the red"; "H_β bright on absorption": "H_β unsymmetrical double bright on wide absorption, the violet bright component is the stronger." Only one statement is made about H_γ, namely, "bright on broad absorption, probably very unsymmetrical double bright, with the violet component much the stronger."

H.D.C. 193237 (P Cyg.) $20^h\ 14^m{\cdot}1$ $+37°\ 43'$ Mag. 4·88 Sp. B1p.

Negative No. 339 1923 Sept. 5 $8^h\ 40^m$

Nine spectra of this star were taken in 1922 and two spectra in 1923 September and December. They all exhibit practically identical features with regard to the hydrogen lines. This star is not included in Merrill's programme, but the spectrum is here briefly described to record its present appearance in case of possible changes later. The hydrogen lines, as far as H_θ, are all double, consisting of bright red components and dark violet components. The helium lines are of a similar nature and very strong. The spectrum contains also numerous other pairs of bright and dark lines of considerable intensity. [See Plate 9, No. 4.]

H.D.C. 200120 (f' Cyg.) $20^h\ 56^m{\cdot}4$ $+47°\ 8'$ Mag. 4·86 Sp. B0p.

Negative No. 344 1923 Sept. 7 $9^h\ 2^m$

Only two photographs of the spectrum of this star were secured, and the better of the two stated above was only moderately satisfactory.

This shows H_β as possibly a single bright line on a broad absorption band, and H_γ and H_δ as certainly single bright lines on broad absorption bands.

Merrill's observations of 1911 and 1912 describe H_β as "possibly bright on absorption" and "indistinctly bright," while his remarks in the case of H_γ are "probably bright on absorption."

It may be remarked here that Miss Cannon, from photographs taken at the end of 1904, described the hydrogen lines from H_β to H_ζ as bright and superposed on wide, hazy, dark bands. Since then the bright lines have gradually been reduced in intensity until they are now nearly on the point of extinction.

A full summary of the remarkable changes in this spectrum from the earliest observations down to the year 1921 has recently been published by R. H. Curtiss.*

* *Pub. Astr. Obs. Univ. Michigan*, 3, p. 19.

H.D.C. 202904 (v Cyg.) $21^h\ 13^m{\cdot}8 + 34^\circ\ 29'$ Mag. 4·42 Sp. B3p.

Negatives No.	327	1923 Aug. 7	10^h	39^m
	383	,, Oct. 31	8	9

Five negatives were taken between 1923 August 1 and November 1, of which the best is No. 383 above.

This shows all the hydrogen lines down to H_η as double bright lines on broad absorption bands, and in all cases as far as H_ϵ the violet component is stronger than the red. On negative 327 the bright line in H_β is not separated, but this may be due to less good definition: H_γ and H_δ appear as double bright and the components seem to be of equal density. H_ϵ is bright but doubtfully double bright: H_ζ appears as a narrow absorption line on a broad absorption band. These differences may all be probably due to the better definition of negative 383.

Merrill's observation of 1911 and 1912 indicate that in 1911 H_β was "bright on absorption," and that in 1912 H_β was "double bright on weak absorption" in August and "bright on absorption" in June. On two other negatives he describes H_β and H_γ as "double bright on diffuse absorption."

The Sidmouth photographs suggest that the bright lines must have become very much more intense. [See Plate 9, No. 6.]

H.D.C. 209409 (o Aqr.) $21^h\ 58^m{\cdot}1 - 2^\circ\ 38'$ Mag. 4·66 Sp. B5p.

Negative No. 340 1923 Sept. 5 $9^h\ 56^m$

Only one negative of this star was taken.

It shows H_β as a strong double bright line on a broad absorption band with the components of about equal intensity. H_γ is also double bright on a broad absorption band, but the bright lines seem to be detached from the narrow absorption line between them. H_δ and H_ϵ indicate narrow absorption lines on broad absorption bands with a suggestion of bright borders to the narrow absorption line in H_δ.

Merrill's observations in 1912 describe H_β as "distinct double bright on absorption" and H_γ as "broad absorption, possibly with bright borders."

H.D.C. 212076 (31 Peg.) $22^h\ 16^m{\cdot}6 + 11^\circ\ 42'$ Mag. 4·93 Sp. B3p.

Negative No.	360	1923 Sept. 11	9^h	24^m
	385	,, Nov. 11	8	42

Three negatives were secured during the months 1923 September and November. No. 385 is a little denser than 360 mentioned above.

Both negatives show H_β, H_γ, H_δ, and H_ϵ as single bright lines on broad absorption bands, but on negative 385 it is very probable that in the case of H_β and H_γ the bright lines are really close doubles.

Merrill in his observations of 1909 to 1911 describes H_β as "bright on absorption" and H_γ as "double bright on absorption." From two plates taken in 1911 and 1912 he describes H_β as "bright on absorption" and "bright on wide rather strong absorption," but does not mention H_γ.

H.D.C. 212571 (π Aqr.) $22^h\ 20^m{\cdot}2 + 0°\ 52'$ Mag. 4·64 Sp. B1p.

Negative No. 367 1923 Sept. 16 $9^h\ 55^m$

Three negatives were secured during the months 1923 August and September, of which that above was the most satisfactory. H_β, H_γ, and H_δ are all well-defined double bright lines on broad absorption bands, and in all cases the violet component is stronger than the red. H_ϵ may possibly be weak double bright on broad absorption, but it is not distinct.

The spectrum appears to be a weak edition of γ Cassiopeiæ. Merrill's observations of 1912 show that H_β was double bright and that H_γ was a "wide double bright line but too poor to measure." About another negative he says: "Bright H_β is unsymmetrical with the violet component the stronger." He continues further, and states "this may possibly be due to photographic distortion, which may explain this discordance of the measured separations."

H.D.C. 217891 (β Pis.) $22^h\ 58^m{\cdot}8 + 3°\ 17'$ Mag. 4·58 Sp. B5p.

Negative No. 329 1923 Aug. 7 $13^h\ 1^m$
361 „ Sept. 11 10 33

The above were the only two negatives taken and both are very satisfactory. They show H_β and H_γ as possessing strong single bright lines symmetrically situated on broad absorption bands. They are probably very close doubles. H_δ is clearly double on both negatives, but H_ϵ appears double in negative 329 but probably only single in negative 361. In negative 329 H_ζ is single bright on broad absorption and H_η questionably bright on broad absorption.

Merrill's observations of 1911 and 1912 show that on all his plates he recorded H_β and H_γ sharp bright on very strong absorption, but they are not separated enough for measurement. No mention is made at all of the other hydrogen lines.

The following table presents, in a very abbreviated form, a summary of the appearances of the hydrogen lines in the stars mentioned in the previous paragraphs.

The abbreviations used are as follows:—

A	Absorption.
DB on A	Double bright on Absorption. A double bright line superposed symmetrically on a broad absorption band, *i.e.* absorption visible outside the bright lines.
SB on A	Single bright on Absorption, symmetrically situated.
BA or NA	Broad Absorption or Narrow Absorption.
V or R	Violet or Red component of a double bright line.
V>R or R>V	Violet or Red component more intense than a Red or Violet component respectively.
DV or DR	The double or single bright lines are not symmetrically situated on the broad absorption, but displaced towards the violet or red edges respectively.

The numbers in the first column are the Henry Draper Catalogue star numbers.

TABLE.

Descriptions of the Individual Hydrogen Lines.

Star.	Negative No.	H_β.	H_γ.	H_δ.	H_ϵ.	Other Remarks.
5394	356	DB on NA	DB on BA	DB on BA	DB on BA	$H\zeta$, DB on BA, V>R.
(γ^1Cas.)		V=R	V=R	V>R?	V>R	$H\eta$, DB on A, V=R.
	414	DB on NA	DB on BA	DB on BA	DB on BA	$H\zeta$, DB on BA, V>R.
		V=R	V=R	V>R	V>R?	
10516	355	DB on NA, DR	DB on BA, DR	DB on BA	SB on BA	$H\zeta$, DB, R>V.
(ϕ Per.)		R>V	R>V	R>V	R	$H\eta$, DB, R>V.
	403	DB on BA, DR	DB on BA	DB on BA	SB on A	$H\zeta$, DB, R=V.
		R>V?	R=V	R=V	R>V	$H\eta$, DB, R>V.
	435	DB on BA, DR	DB on BA	DB on BA	DB on A	$H\zeta$, DB, V>R?
		V>R?	V>R	V>R?	V>R	$H\eta$, NA on BA.
20336	357	DB on BA	DB on BA	DB on BA	DB on BA	$H\zeta$, DB on BA.
(−Cam.)		R>V	R>V	R>V	R>V	
22192	364	DB on BA, DR	DB on BA	DB on BA	SB on BA	$H\zeta$, NA on BA.
(ψ Per.)		V=R	V=R	V=R	R	
	427	DB on BA, DR	DB on BA	DB on BA	SB on BA	$H\zeta$, NA on BA.
		V=R	V=R	V=R	R	$H\eta$, NA on BA.
	436	DB on BA	DB on BA	DB on BA	NA on BA	$H\zeta$, SB on BA.
		V>R	V>R?	V>R?		$H\eta$, SB on BA.
23302	370	SB on BA, DR	SB on BA, DR	SB on BA?	BA	
(17 Tau.)	422	D(?)B on BA	D(?)B on BA	D(?)B on BA	SB on BA	
	430	DB on BA	DB on BA, DR R>V	D(?)B on BA	DB on BA	
23480	370	DB on BA	D(?)B on BA	NA on BA	BA	
(23 Tau.)	399	DB on BA	D(?)B on BA	BA	BA	
23630	348	DB on BA	NA on BA	NA on BA	NA on BA	
(η Tau.)		V=R				
	370	DB on BA	SB on BA, DR	SB on BA	D(?)B on BA	$H\zeta$, NA on BA.
		V>R?				
	399	DB on BA	NA on BA	NA on BA	BA	
		V=R				
23850	370	SB on BA	SB on BA	SB on BA	BA	
(27 Tau.)	422	SB on BA	SB on BA	SB(?) on BA	BA	

TABLE—*continued.*

Star.	Negative No.	H_β.	H_γ.	H_δ.	H_ϵ.	Other Remarks.
25940	365	SB on BA	DB on BA	DB on BA	DB on BA	Hζ, DB on BA, R>V.
(c Per.)			R>V	R>V	R>V	
	409	DB on BA	DB on BA	DB on BA	D(?)B on BA	Hζ, D(?)B on BA.
		R>V	R>V	R>V		
35439	416	DB on BA, DR	DB on BA, DR	SB on BA, DR	SB on BA	
(25 Ori.)		V>R	V>R			
37202	265	DB, V>R	NA	NA	NA	
(ζ Tau.)	351	DB, V=R	NA	NA	NA	
	400	DB, V>R	NA	NA	NA	
45542	439	DB on BA	NA on BA	BA	BA	
(ν Gem.)		V>R ?				
109387	46	DB on BA	DB on BA	NA on BA	NA on BA	1915 Photograph.
(κ Dra.)		R>V ?	R>V			
	50	DB on BA	DB on BA	DB on BA	D(?)B on BA	1915 Photograph.
		R>V	R>V			
191610	382	DB on BA	DB on BA	NA on BA	BA	
(b^2 Cyg.)		V>R	V>R			
	384	DB on BA	DB on BA	BA	BA	
		V>R ?				
193237	339	Bright and	Bright and	Bright and	Bright and	Hε, Hζ, Hη, Hθ,
(P Cyg.)		Dark Pairs	Dark Pairs	Dark Pairs	Dark Pairs	Bright and Dark Pairs. Bright, red side; dark, violet side.
200120	344	SB(!) on A	SB on BA	SB on BA	BA	
(f' Cyg.)						
202904	327	D(?)B on BA	DB on BA	DB on BA	D(?)B on BA	Hζ, NA on BA.
(υ Cyg.)			V=R	=V	R	
	383	DB on BA	DB on BA	DB on BA	DB on BA	Hζ, DB on BA.
		V>R	V>R	V>R	V>R	Hη, DB on BA.
209409	340	DB on BA	DB on BA	NA on BA	NA on BA	
(o Aqr.)		V=R				
212076	360	SB on BA	SB on BA	SB on BA	SB on BA	
31 Peg.)	385	D(?)B on BA	D(?)B on BA	SB on BA	SB on BA	
212571	367	DB on A	DB on BA	DB on BA	B(?) on BA	
(π Aqr.)		V>R	V>R	V>R		
217891	329	SB on BA	SB on BA	DB onBA	DB on BA	Hζ, B on BA; Hη, B(?) on BA.
(β Pis.)	361	SB on BA	SB on BA	DB on BA	B(!) on BA	

Description of the Plate.

The spectra here illustrated show eight of the stars which exhibit bright hydrogen lines. That placed at the top, γ Orionis, is a star of type B2 with absorption lines only to serve as a useful comparison star for scale.

As the original spectra are on the average about 3·5 inches long and about one-twentieth of an inch broad they are not suitable for reproduction on account of their small width. They have been broadened in the following manner:—

A clepsydra apparatus, designed by the writer, after the principle of the South Kensington Solar Physics Observatory's spectroheliograph apparatus, was constructed by Messrs Cooke, Troughton, & Simms, Ltd. The original negatives were enlarged on imperial process plates and at the same time broadened by means of the clepsydra. The positives thus produced were then again slightly enlarged on similar plates and the scale of enlargement so adjusted as to bring all the spectra to the same scale. From these negatives contact prints on velox paper were made and a narrow strip of the whole spectrum was cut out, ensuring the perpendicularity of the lines to the direction of dispersion. The scale of reproduction of the plate is slightly less than twice the original scale of the negatives.

The following spectra are reproduced, accompanied by their spectral types and dates of exposure:—

1.	γ Orionis	B2	1923 Dec. 13
2.	ϕ Persei	Bop	,, Sept. 11
3.	γ Cassiopeiæ	Bop	1921 Sept. 30
4.	P Cygni	B1p	1923 Sept. 5
5.	ζ Tauri	B3p	,, Sept. 7
6.	ν Cygni	B3p	,, Oct. 31
7.	c Persei	B3p	,, Dec. 15
8.	η Tauri	B5p	,, Sept. 7
9.	ψ Persei	B5p	,, Sept. 11

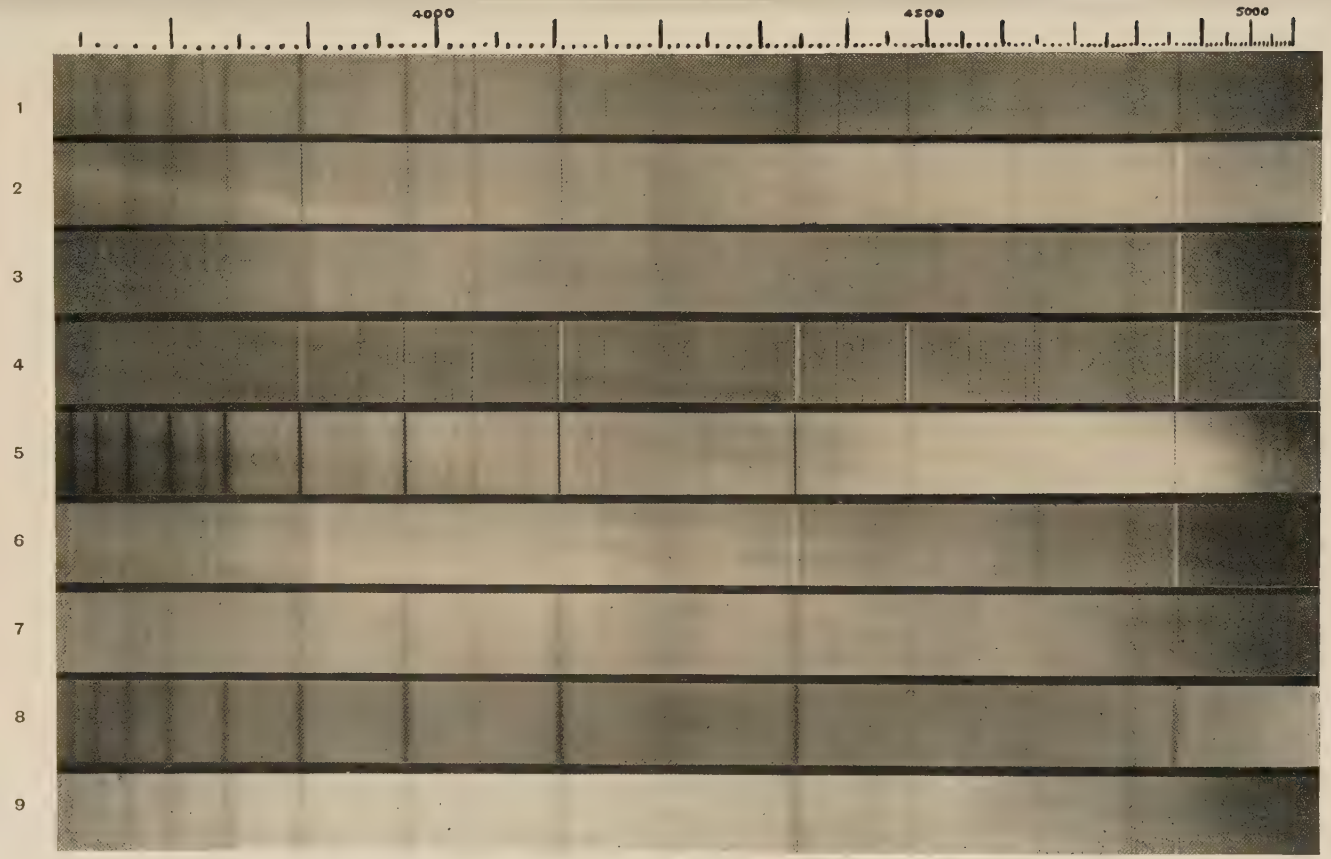
Summary.

1. This research is confined to recording the appearances of the hydrogen lines during the winter of 1923-24 in the spectra of stars which have bright hydrogen lines.

2. It may be considered a re-survey of the spectra of many of the stars which were examined from this point of view by Dr. Paul W. Merrill with the 36-inch Lick Refractor during the year 1911 to 1912.

3. Twenty-one individual stars have been dealt with, involving a total exposure time of 111·6 hours, giving an average exposure time for each plate of 52·1 minutes.

4. All the photographs were taken with one instrument, namely, the 9-inch Prismatic Camera of the Norman Lockyer Observatory,



1. γ ORI. B2. 2. ϕ PER. Bop. 3. γ CAS. Bop. 4. P. CYG. B1p. 5. ζ TAU. B3p. 6. υ CYG. B3p. 7. c PER. B3p. 8. η TAU. B5p. 9. ψ PER. B5p.

with a prism of 45° angle and a dispersion of 46·0 mm. between H_β and H_δ.

5. Details of the appearances of the lines, together with Merrill's results, are given in a series of separate sections under the headings of each star.

6. A table is given summarising in an abbreviated form the recorded appearances of the hydrogen lines.

7. A list of stars is added representing those which exhibit the more interesting cases in which the present research has revealed notable changes in the appearances of the hydrogen lines.

8. A plate accompanies the text, illustrating eight of these bright hydrogen line stars.

Effective Wave-length and Spectral Classification of Faint Stars. Standards of Reference.

By C. R. Davidson and E. Martin.

(*Communicated by the Astronomer Royal.*)

In a paper on the "Effective Wave-lengths of Stars in relation to Spectral Type" (*M.N.*, 1921 Dec.), it was suggested that effective wave-length provides a practical method for determining the spectral type or colour of very faint stars.

In the course of the work it became apparent that to realise its full usefulness it would be necessary to adopt some standard of reference to which all observers might correlate their results.

The wave-length of maximum photographic intensity will vary with the—

(1) Telescope employed—refractor or reflector.
(2) Photographic plate.
(3) Density of the image.
(4) Quality of the sky.
(5) Altitude of the star, etc.

In *Ast. Nach.* 5159, Bergstrand and Rosenberg point out the convenience of the grating and its suitability for a colour Durchmusterung for faint stars, and publish a list of 25 stars near the North Pole, of spectral type ranging from B to M, and ask that they should be observed by many observers with different instruments.

These stars may then be used to serve as standards in the same way as the North Polar Sequence is used for standards of magnitude. In this way the various errors enumerated above will be eliminated, and the colours of the stars determined with great accuracy.

The list of stars proposed by Bergstrand is given in Table I.

TABLE I.

Standard Stars.

No.	B.D.	R.A. 1900·0.	Dec. 1900·0.	Magnitude. Photog.	Magnitude. Visual.	Colour Index.	Spectral Type.
	°	h m s	° ′	m	m	m	
1	81 13	0 32 13	+81 56·5	6·87	6·25	+0·62	F_5
2	78 34	1 3 36	+79 8·5	5·32	5·59	−0·27	B_9
3	80 64	1 57 4	+80 49·1	6·03	5·95	+0·08	A_2
4	78 103	2 52 47	+79 1·4	7·25	5·38	+1·87	M
5	84 59	3 8 34	+84 33·5	6·38	5·45	+0·93	G_0
6	86 51	3 33 54	+86 20·0	6·13	5·78	+0·35	F_0
7	83 104	4 4 59	+83 33·9	5·04	5·50	−0·46	B_6
8	80 155	4 41 37	+81 1·7	6·47	5·06	+1·41	K_4
9	85 78	5 9 52	+85 35·3	6·53	6·64	−0·11	A_0
10	82 201	7 10 4	+82 36·3	6·62	4·90	+1·72	M
11	81 282	8 56 18	+81 13·8	6·76	6·27	+0·49	F_1
12	83 297	10 18 55	+83 4·1	5·39	5·07	+0·32	A_8
13	78 412	12 7 31	+78 10·3	5·40	5·20	+0·20	A_4
14	83 431	14 57 3	+82 55·4	6·20	5·49	+0·71	F_8
15	77 699	18 34 35	+77 28·1	6·92	5·59	+1·33	K_2
16	83 536	18 37 22	+83 6·1	6·14	6·17	−0·03	A_0
17	77 764	20 12 16	+77 24·6	4·22	4·63	−0·41	B_5
18	80 659	20 34 32	+81 4·8	6·53	5·48	+1·05	G_4
19	80 660	20 35 15	+80 44·5	6·97	5·97	+1·00	G_2
20	78 716	20 39 57	+79 4·6	6·65	6·97	−0·32	B_2
21	80 672	20 52 8	+80 10·6	6·67	5·49	+1·18	G_8
22	79 707	21 27 46	+80 5·4	6·79	6·01	+0·78	G_1
23	85 383	22 21 18	+85 36·3	5·06	5·24	−0·18	B_8
24	82 703	22 47 53	+82 37·4	6·12	4·62	+1·50	K_2
25	83 640	22 55 12	+83 48·7	6·19	4·57	+1·62	K_6

It will be noticed that this list comprises stars of all spectral types from B to M, that the stars are of 5th or 6th magnitude, and being circumpolar are suitable for standards in the northern hemisphere. This programme was adopted at Greenwich, and with the 30-inch reflector stopped down to 20 inches, and a wire grating of equal bar and space and of approximately 3·0 mm. pitch, a number of photographs were accordingly taken of each star, several stars on each plate. Two sets of exposures were given in the ratio of 1 : 3, so that the intensities differed by 1 magnitude. The length of exposure was such that one set was fainter and the other brighter than the adopted standard intensity, *i.e.* 10 minutes' exposure on a 10th magnitude star. The mean was corrected to this standard, and the results for each night are given in Table II.

TABLE II.

Measured Effective Wave-length of Standard Stars.

Seq. No.	July 26.	July 28.	Aug. 2.	Aug. 4.	Aug. 8.	Oct. 22.	Dec. 19.	Dec. 23.	Jan. 6.	Jan. 20.	Mar. 9.	Mar. 11.
	1923.								1924.			
1	4276	4281	4262									4296
2	4224	4244	4234									4254
3	4245		4230									4258
4			4544	4570	4572						4582	
5			4364	4418	4408						4400	
6			4252	4288	4270						4268	
7			4148	4150	4150							
8			4494	4534	4511							
9						4264		4291	4253			4244
10							4560	4548	4548		4542	4534
11							4317	4330	4268		4263	4271
12							4300	4324	4274		4254	
13	4265	4274	4272									
14			4304	4356	4343							
15						4511		4518	4493			
16						4267		4298	4258			
17								4269	4236	4287		
18								4480	4462	4488		
19								4460	4431	4474		
20								4142	4109	4179		
21								4498	4476	4512		
22								4466	4424	4453		
23								4262	4228	4269		
24	4464	4488	4498									
25	4521	4536	4524									

Observation extended from 1923 July 26 to 1924 March 11, the completion being delayed by persistent bad weather and pressure of other work.

Several stars of different colours were observed on each night, and it was arranged on subsequent nights that some of these should be observed with other stars to provide for final reduction to uniform conditions. The stars were observed at two main periods—1923 July 6 to August 8, and 1923 October 22 to 1924 January 20—which were connected by observations on 1924 March 9 and 11.

A correction to the night was obtained by comparing the values obtained on successive nights, and the final result is reduced to the mean conditions of the 12 nights, and is in sensible accordance with that obtained in the regular work on the Astrographic Catalogue.

The residuals derived from a comparison of the separate results with the adopted mean value are shown in Table III.

TABLE III.

Residuals for Separate Nights.

Seq. No.	July 26.	July 28.	Aug. 2.	Aug. 4.	Aug. 8.	Oct. 22.	Dec. 19.	Dec. 23.	Jan. 6.	Jan. 20.	Mar. 9.	Mar. 11.
	1923.									1924.		
1	−31	−26	−45									−11
2	−43	−23	−33									−13
3	−28		−43									−15
4			−37	−11	− 9						+ 1	
5			−47	+ 7	− 3						−11	
6			−31	+ 5	−13						−15	
7			−16	−14	−14							
8			−34	+ 6	−17							
9						+10		+37	− 1			−10
10							+22	+10	+10		+ 4	− 4
11							+35	+48	−14		−19	−11
12							+25	+49	− 1		−21	
13	−39	−30	−32									
14			−45	+ 7	− 6							
15						+20		+27	+ 2			
16						+ 9		+40	0			
17								+33	0	+51		
18								+31	+13	+39		
19								+33	+ 4	+47		
20								+27	− 6	+64		
21								+31	+ 9	+45		
22								+46	+ 4	+33		
23								+37	+ 3	+44		
24	−53	−29	−19									
25	−40	−25	−37									

The means of the residuals for the separate nights are—

1923	July 26	−39 t.m.
	28	−27
	Aug. 2	−35
	4	0
	8	−10
	Oct. 22	+13
	Dec. 19	+27
	23	+35
1924	Jan. 6	+ 2
	20	+46
	Mar. 9	−10
	11	−11

There is a range amongst the summer nights of 39 t.m. and the winter nights of 44 t.m., and a total range of 85 t.m. The total range is certainly larger than was expected, as the photographs were taken only on apparently fine nights, but when the separate residuals are

examined in relation to wave-length the absorption does not appear to affect the blue more than the red, at any rate within the limits with which we are dealing, and a single correction for zero only is required to bring the separate nights to a common standard.

As to the causes, it should first be noted that the mirror was newly silvered in 1923 July, and was in good condition for July and August, in very fair condition for December and January, and the March plates were taken with the mirror untouched. The positive residuals of December and January cannot be accounted for by the state of the mirror.

The want of transparency of the winter sky may be due to the greater amount of smoke present in winter, or perhaps to greater humidity. A comparison with the hygrometer is inconclusive, although it does appear that July 26 is the driest night with 61 per cent. humidity, and January 20 is the wettest night with 86 per cent., but the other dates do not support this hypothesis. The effect of humidity on the plate is a possible factor, as the plate is freely exposed to the damp air in the mouth of the telescope tube.

The concluded effective wave-lengths for the average of the 12 nights are given in Table IV.

TABLE IV.

Effective Wave-length of Standard Stars.

Serial No.	No. in B.D.	Photog. Mag.	Effective Wave-length.	Colour Index.	Spectral Type.	Colour Index. (Parkhurst).	Spectral Type. (Parkhurst).	Spectral Type (Harvard).
	°	m	t.m.	m		m		
20	78 716	6·6	4115	− ·76	B1	− ·32	B2	B3
7	83 104	5·0	4164	− ·49	B4	− ·46	B6	B3
23	85 383	5·1	4225	− ·14	B8	− ·18	B8	A0
17	77 764	4·2	4236	− ·08	B9	− ·41	B5	B9
9	85 78	6·5	4254	+ ·02	A0	− ·11	A0	A0
16	83 536	6·1	4258	·04	A1	− ·03	A0	A2
2	78 34	5·3	4267	·10	A3	− ·27	B9	A0
3	80 64	6·0	4273	·13	A6	+ ·08	A2	A0
12	83 297	5·4	4275	·15	A7	·32	A8	F5
11	81 282	6·8	4282	·18	F2	·49	F1	F2
6	86 51	6·1	4283	·19	F2	·35	F0	F5
13	78 412	5·4	4304	·30	F7	·20	A4	A5
1	81 13	6·9	4307	·32	F7	·62	F5	F8
14	83 431	6·2	4349	·56	G2	·71	F8	G0
5	84 59	6·4	4411	·91	G6	·93	G0	K0
22	79 707	6·8	4420	·96	G6	·78	G1	K0
19	80 660	7·0	4427	1·00	G7	1·00	G2	K0
18	80 659	6·5	4449	1·12	G8	1·05	G4	K0
21	80 672	6·7	4467	1·22	K0	1·18	G8	K0
15	77 699	6·9	4491	1·35	K1	1·33	K2	K0
24	82 703	6·1	4517	1·50	K3	1·50	K2	K0
8	80 155	6·5	4528	1·56	K4	1·41	K4	K0
10	82 201	6·6	4538	1·62	K5	1·72	M	Ma
25	83 640	6·1	4561	1·75	K8	1·62	K6	K0
4	78 103	7·2	4581	1·86	Ma	1·87	M	Ma

The colour index in column 5 is derived from the formula $C = \frac{\lambda - 4250}{178}$, and the spectral type in column 6 is taken from a curve slightly modified from that given in *M.N.*, 1921 December, as shown in Table V.

TABLE V.

Effective Wave-length and Equivalent Spectral Type.

4105	B0	4320	G0
4186	B5	4396	G5
4250	A0	4474	K0
4270	A5	4540	K5
4278	F0	4580	Ma
4292	F5		

Parkhurst's colour index and spectral type are from Yerkes Photometry (*Astrophys. Jour.*, 1912 October), and the Harvard spectral type from the Draper Catalogue.

These results are put forward as providing provisional standards, and it is suggested that in adopting definitive standards the following points should be considered:—

(i) The reflecting telescope should be the standard telescope, because of its freedom from chromatic aberration and the consequent absence of error arising from focussing and distance from the optical axis. A discussion of results with the 30-inch reflector shows that effective wave-lengths may be determined with accuracy when the star is more than 60′ from the centre of the field, if allowance is made for the relative intensity of the image when near the centre of the field and when at a distance from it.

(ii) The density of the standard image should be defined. The standard image adopted here is that of a 10th mag. star with an exposure of 10 minutes when the 30-inch reflector is stopped down to 20 inches, giving an aperture ratio of 1 : 6·75. This image is not opaque, and the point of maximum intensity is easily distinguishable.

(iii) Effective wave-length should be corrected to the zenith. The photographs on which these results depend were taken at the altitude of the Pole at Greenwich. Adopting Hertzsprung's value of selective atmospheric absorption, which at sea-level and Z.D. 60° causes an increase in effective wave-length of 35 t.m., these should be corrected by $-35\ (\text{sec. } 38^\circ\ 31' - 1) = -10$ t.m.

(iv) The quality or transparency of the sky should be checked whenever observations are made.

This is easily done by exposing on the field near the North Pole as is done when determining photographic magnitudes. The measures can then be corrected to an adopted value. It is the practice at Greenwich that whenever photographs are taken of the astrographic fields, one plate at least shall receive two exposures of 3 minutes' duration on the polar area. A list of stars within one degree of the Pole with their effective wave-lengths is given in Table VI.

TABLE VI.

Effective Wave-length of Stars in the Polar Area.

No. in B D.		Photog. Mag.	Effective Wave-length.	Harvard Spectral Type.
88°	2	8·7	4402	Ko
89	2	10·2	4528	
	3	9·0	4261	A5
88	10	9·6	4418	
89	6	10·0	4292	
	7	9·6	4250	
	9	10·0	4484	
	12	10·1	4315	
	13	7·1	4256	Ao
	15	9·8	4362	
	17	9·4	4279	
	18	9·6	4271	
	21	9·3	4277	F
	22	9·9	4512	Ko
	25	9·9	4286	
	28	9·7	4535	
	30	9·8	4253	
88	112	7·9	4526	Mb
	131	8·7	4260	A5
89	38	9·5	4255	

The effective wave-lengths given here have been determined from a number of plates, but it would be useful to have the determinations repeated at another observatory.

The Rotation Period of the Sun Derived from Measures of Solar Faculæ made at the Royal Observatory, Greenwich. (Plate 10.)

(*Communicated by the Astronomer Royal.*)

Introduction.—The determination of the Sun's rotation period from observations of faculæ involves difficulties which are not inherent in the measures of sun-spots or calcium flocculi. Firstly, the faculæ do not offer definite and relatively stable points for measurement such as are afforded by the spots, and secondly they cannot be traced across the Sun's disc in the same way as the spots and flocculi. Being visible by ordinary photographic processes only near the Sun's limb, the error of measurement on account of foreshortening is approaching a maximum, and individual features in a group of faculæ can seldom be traced for longer than two or three days. They possess, however, one definite advantage over the sun-spots. Although individual features do not

persist for long, groups of faculæ, such as surround sun-spots, are longer-lived than the spots which they accompany. Moreover, they tend to disappear as a whole, whilst in a large proportion of spot groups the disappearance of the following spots takes place some time before the extinction of the leading ones. It appeared probable, therefore, that a selection of the smaller, more definite and persistent areas of faculæ for measurement as entities would offer rather better material, with their intervals of about 27 or 54 days, than a selection of definite points at intervals of about one or two days.

Material Employed.—In order to supply material for a recent paper on solar faculæ (*M.N.*, *R.A.S.*, 84, 96), the life history was traced for several hundred groups. Of these, a number were followed for two or three rotations and a few for longer periods. These recurrent faculæ had been tabulated for the years 1910–1916, and 1920–1922, and the list was subsequently extended to include other years,—1888, 1898–1903, 1917 (part), 1919, and 1923. The examination was chiefly restricted to years of lesser solar activity. When the faculæ zones are fully active, it is usually impossible to define, after a solar rotation, the exact extent of a group owing to neighbouring disturbances, and the identification is necessarily most certain when the areas are small and isolated.

Each recurrent series of faculæ was then examined in detail from the original photographs, and a selection was made which included only those groups which fulfilled the following conditions: (1) Definite appearance, (2) small or moderate extent, (3) sufficiently isolated from other groups to ensure correct identification in succeeding rotations, (4) unbroken activity, as far as could be judged from their successive appearances at the Sun's limbs. Such a selection was made entirely from the photographs and not from the results of the measures of position, though necessarily the approximate positions entered into the identification. All faculæ so included in this selection were retained, whatever rotation periods were derived from their measures. The number of faculæ which lasted to the second and third rotations and fulfilled the above requirements were not numerous, and nearly all of them proved to be associated in their early history with sun-spots. In all 197 areas were selected; of these, 140 were seen in the second rotation, 53 in the third, and 4 in the fourth rotation.

Method of Reduction.—The fullest determinations hitherto published of the solar rotation period derived from faculæ are those of Stratonoff from Poulkovo photographs, 1891–1893, and those of Chevalier from Zô-sè photographs taken between 1905 and 1908. The measures on which both determinations are based depend upon the identification of individual features traced for successive days. In Stratonoff's determination, 954 of 1062 comparisons had an interval of one day; 103 of 2 days, and the remaining 5 comparisons of 3 days intervals. The Zô-sè determination depended upon 2571 comparisons. The corresponding intervals are not stated, but presumably they are similar to Stratonoff's.

The present investigation, as already stated, is based upon comparisons made at intervals of a solar rotation. There is, of course, the uncertainty of identification (with the possibility of including an adjoining

disturbance originating on the invisible hemisphere or even during the transit on the visible hemisphere), combined with the error of measuring near the Sun's limb the centre of an area of some extent instead of a point, but judging from the mean deviation from the curve, these disadvantages are far outweighed by the much longer intervals between the observations. One is, moreover, concerned with groups of faculæ as units and not with separate parts, which may well have proper motions of their own. The uncertainty of identification is reduced to a minimum by careful examination of the photographs on which the shape, brightness, and other characteristics of the faculæ can be noted. The error of measurement due to foreshortening is to a large extent eliminated by the simple device of pairing the observations, separated one rotation apart, according to their longitude, from the Sun's central meridian.

A number of the measures of position were obtained from the published results in the *Greenwich Photoheliographic Results*, but a large proportion of the selected areas of faculæ required re-measurement chiefly on account of their association with sun-spots. In the *Results* the positions of faculæ associated with spots are given only relatively to them—whether they precede, follow, etc. In all, about 2000 measures were involved.

A tabulation for each group of faculæ was then made as follows:—

Date 1922.	Day of Year and Decimal.	Longitude.	Latitude.	Longitude from Central Meridian.
		°	°	°
Mar. 2	60·43	40·4	+ 9·6	−72·2
3	61·10	39·6	+ 9·3	−64·2
4	62·57	38·1	+ 8·3	−46·3
11	69·42	40·6	+ 7·1	+46·5
12	70·42	39·2	+ 9·3	+58·3
13	71·34	39·5	+ 8·0	+70·7
29	87·33	43·9	+ 8·5	−74·1
30	88·37	42·5	+ 9·2	−61·8
31	89·12	43·2	+11·7	−51·2
Apr. 7	96·37	44·0	+ 8·0	+45·3
8	97·35	41·7	+10·1	+55·9
9	98·53	40·6	+ 6·6	+70.4

From these positions of longitude, the rotation periods from each pair of points were derived in the following table:—

Date. From.	Date. To.	Interval in Days = y.	Change in Longitude = x	x/y.	Sidereal Rotation Period.	Mean Latitude.
			°		Days.	
Mar. 2	Mar. 29	26·90	+3·5	+·13	25·15	
3	30	27·27	+2·9	+·11	25·18	
3–4	31	27·04	+4·6	+·17	25·08	
11	Apr. 7	26·95	+3·4	+·13	25·15	+8·8
12	8	26·93	+2·5	+·09	25·22	
13	9	27·19	+1·1	+·04	25·31	
				Mean	25·18	

The respective sidereal rotation periods were derived by means of a table in which were tabulated the values of $\frac{x}{y}$ from $+\cdot 80$ to $-1\cdot 10$ with the corresponding rotation periods computed from the expression

$$\frac{25\cdot 38}{1+\frac{x}{y}\cdot\frac{25\cdot 38}{360}}$$

where 25·38 is the sidereal rotation period, according to Carrington, as adopted in the ephemeris given in the *Nautical Almanac*, and on which the computations of longitude are based.

A complete list of the deduced mean sidereal rotation periods, as given by each group of faculæ, is shown in the following table. These are also plotted in the accompanying diagram. In deriving the mean period from the separate observations given by faculæ lasting to the 3rd and 4th rotations, equal weights were assigned to the periods derived from 1st to 2nd, 1st to 3rd, and 1st to 4th. Almost without exception, the faculæ selected for measurement were compact and definite during the first rotation, and became increasingly scattered and faint in the succeeding rotations. It was therefore considered that the longer period compensated for the increased uncertainty of measurement.

Date.			Mean Latitude.	Sidereal Rotation Period.	No. of Observations.	Extreme Range.	Remarks.
Year.	First Measured on	Interval to Last Measure.					
		Days.	°	Days.		Days.	
1888	Jan. 10	38	0·0	24·78	5	0·51	
	10	38	− 8·8	24·88	6	0·40	
1890	Feb. 14	31	−32·0	26·24	3	0·38	
	Mar. 4	39	+33·8	26·56	4	0·66	
	July 5	38	−24·7	25·50	4	0·23	
	23	11	−34·9	26·23	3	..	*
	23	39	− 6·9	25·56	4	0·26	
	Sept. 6	39	+25·7	25·43	3	0·14	
	Oct. 1	59	+24·1	25·72	9	0·20	
	1	58	+26·0	25·67	6	0·39	
	Nov. 8	38	−24·9	25·47	3	0·17	
	16	66	−24·0	25·52	8	0·27	*p*
	Oct. 29	50	−25·1	25·68	6	0·19	*f*
1898	Jan. 20	37	+ 5·1	24·94	4	0·47	
	21	37	+ 4·4	24·92	4	0·20	
	May 8	37	− 6·6	24·79	6	0·52	
1899	Mar. 2	46	− 4·6	25·09	5	0·43	
1900	May 4	54	+ 5·2	24·96	7	0·86	
	Sept. 4	38	− 5·3	24·86	6	0·24	

Date. Year.	First Measured on	Interval to Last Measure.	Mean Latitude.	Sidereal Rotation Period.	No. of Observations.	Extreme Range.	Remarks.
		Days.	°	Days.		Days.	
1902	Mar. 3	65	+26·9	25·52	10	0·27	
1903	24	57	+25·8	26·04	5	0·47	
	Apr. 3	66	−22·2	25·67	6	0·64	
	27	27	+17·4	24·97	4	0·51	
	June 14	66	−30·8	26·25	9	1·27	
	July 2	64	−19·9	25·03	7	0·19	*p*
	2	66	−22·4	25·49	8	0·20	*f*
	Aug. 4	32	−23·4	25·08	8	0·47	
	19	56	+26·0	25·47	7	0·19	*p*
	20	39	+27·4	25·74	5	0·39	*f*
	Sept. 20	37	−22·4	25·11	4	0·05	
	27	56	+15·7	25·53	6	0·27	
	Oct. 31	66	+18·7	25·42	6	0·42	
	Dec. 8	38	+15·8	25·10	4	0·22	
	8	40	+21·1	25·54	5	0·22	
	29	39	−16·9	25·10	4	0·45	
1910	Jan. 9	37	+ 6·2	25·27	4	0·52	
	11	36	− 4·1	24·83	2	0·02	
	22	38	−12·4	25·30	5	0·26	
	23	55	−23·7	25·67	6	0·09	
	Feb. 2	67	−17·6	25·21	4	0·22	
	Mar. 6	47	+20·6	25·55	3	0·31	
	25	63	+19·8	25·29	7	0·11	*p*
	25	64	+23·9	25·50	7	0·55	*f*
	25	37	− 6·3	24·41	6	0·57	
	Apr. 30	35	+ 4·5	24·62	4	0·51	
	May 4	37	+11·2	25·01	4	0·49	
	8	28	−17·1	25·52	4	0·16	
	June 4	36	− 7·7	24·46	4	0·32	
1912	Sept. 4	63	− 7·6	24·62	11	0·48	
	6	37	− 5·8	24·51	6	0·77	
	10	37	− 8·8	24·64	5	0·45	
	24	37	− 6·0	24·96	6	0·52	
	Nov. 16	63	− 2·5	24·71	8	0·40	
	Dec. 24	37	+26·9	25·46	6	0·99	
1913	Feb. 19	38	+32·2	26·21	6	0·61	
1914	Mar. 14	34	+45·4	28·06	4	0·30	
	Apr. 1	93	+29·2	25·99	8	0·64	

Date. Year.	First Measured on	Interval to Last Measure.	Mean Latitude.	Sidereal Rotation Period.	No. of Observations.	Extreme Range.	Remarks.
		Days.	°	Days.		Days.	
1914	May 2	46	+22·0	25·46	4	0·54	
	June 11	30	−23·0	25·55	3	0·21	
	13	51	−36·0	26·53	5	0·61	
	July 4	36	−34·9	26·47	6	1·14	*p*
	4	38	−39·4	27·23	6	0·75	*f*
	Aug. 14	66	+22·5	25·48	9	0·26	
	Sept. 10	36	−21·4	25·18	4	0·56	
	12	37	−29·1	25·88	4	0·20	
	Oct. 29	56	+22·0	25·45	5	0·20	
	Nov. 1	29	+22·3	25·33	3	0·26	
	26	37	−19·0	25·43	3	0·28	
	30	55	+16·0	25·31	6	0·18	
1915	Jan. 3	75	+25·9	25·63	6	0·41	*p*
	3	30	+27·3	26·39	2	0·17	*f*
	Feb. 22	64	+16·8	24·98	5	0·42	
	28	56	+20·8	25·28	3	0·31	
	Mar. 17	37	+17·4	25·24	3	0·21	
	July 24	38	−27·5	25·87	5	0·15	
	Aug. 11	38	−31·7	26·54	5	0·49	*p*
	11	39	−29·3	26·92	5	0·23	*f*
	15	37	+14·7	24·92	4	0·21	
	24	39	+27·8	25·54	5	0·23	
	27	37	+19·5	25·24	4	0·34	
	27	39	+27·2	25·73	4	0·16	
	29	38	+23·5	25·38	6	0·27	
	31	37	−29·2	26·05	4	0·22	
	Sept. 10	37	+11·5	24·75	4	0·27	
	21	38	+11·9	25·16	4	0·12	
	Oct. 14	37	+24·8	25·21	5	0·44	*p*
	14	29	+27·2	25·71	3	0·09	*f*
	15	36	−17·6	25·08	3	0·18	
	Nov. 8	39	+37·2	26·87	6	0·30	
	14	38	+13·5	25·03	4	0·77	
	19	28	−26·4	25·71	4	0·40	
	Dec. 28	65	−20·2	25·21	11	0·47	
	29	64	+11·0	24·82	10	0·59	
	29	37	+15·9	25·27	4	0·61	
1916	Jan. 5	91	−17·5	24·83	24	0·44	

Date. Year.	First Measured on	Interval to Last Measure.	Mean Latitude.	Sidereal Rotation Period.	No. of Observations.	Extreme Range.	Remarks.
		Days.	°	Days.		Days.	
1916	Jan. 6	39	−24·8	25·72	4	0·09	
	13	37	+23·7	25·66	3	0·29	
	21	37	+15·5	24·96	5	0·08	
	22	37	+21·7	25·29	5	0·38	*p*
	23	38	+24·8	25·77	6	0·55	*f*
	Feb. 11	36	+13·4	24·59	4	0·08	
	Mar. 3	64	+19·3	25·18	8	0·17	
	10	37	+36·8	26·54	4	0·49	
	24	39	−28·9	25·60	5	0·28	
	Apr. 18	38	+28·0	25·59	5	0·28	
	May 14	38	+25·9	25·51	4	0·92	
	Aug. 7	9	+36·3	27·01	4	..	*
	Oct. 4	39	−34·3	25·99	4	0·10	
	30	38	+26·0	25·48	4	0·34	
	Nov. 24	38	+26·6	25·61	4	0·80	
1917	Jan. 31	28	+ 2·4	24·78	2	0·22	
1919	Nov. 25	56	−21·6	25·12	7	0·25	
	Dec. 9	65	+ 5·1	25·01	8	0·66	
1920	Jan. 2	55	−14·5	25·04	4	0·24	
	5	63	+ 9·7	24·90	6	0·12	
	19	63	+ 7·8	25·06	7	0·42	
	Feb. 2	38	−16·5	25·46	4	0·52	
	10	36	+ 5·4	24·72	2	0·14	*p*
	Mar. 8	36	+ 6·1	24·81	2	0·05	*f*
	Feb. 11	38	−22·0	25·06	3	0·26	
	14	64	−17·3	25·09	7	0·18	
	14	66	−21·7	25·36	9	0·48	
	22	64	−14·9	24·76	6	0·20	
	23	38	− 9·9	24·96	3	0·29	
	Mar. 3	36	+16·9	25·31	2	0·68	
	18	37	+13·5	25·09	2	0·07	
	24	64	−10·7	24·82	11	0·33	*p*
	June 11	28	+12·0	24·90	2	0·11	*f*
	Apr. 30	37	+14·9	25·21	5	0·30	
	May 4	37	−11·4	25·06	3	0·28	
	4	36	− 7·0	24·64	3	0·04	
	10	36	+18·1	24·78	4	0·29	
	10	29	+19·0	25·23	3	0·09	

Date. Year.	First Measured on	Interval to Last Measure. Days.	Mean Latitude. °	Sidereal Rotation Period. Days.	No. of Observations.	Extreme Range. Days.	Remarks.
1920	May 30	27	−11·1	24·69	2	0·02	
	June 18	37	− 6·5	25·06	4	0·14	
	29	37	−14·6	25·36	4	0·11	
	July 9	37	− 5·1	25·03	4	0·54	
	19	39	+22·7	25·74	2	0·29	
	Aug. 7	28	+14·1	24·87	2	0·17	
	13	37	−21·7	25·48	3	0·12	
	14	92	+20·3	25·30	12	0·52	
	Sept. 5	37	+10·0	24·82	3	0·33	
	10	37	+ 2·3	24·73	7	0·32	
	28	72	+14·1	24·89	6	0·62	
	Oct. 4	55	+18·2	25·23	5	0·60	
	20	36	+19·7	25·44	2	0·35	
	21	28	−10·5	25·23	2	0·11	
	30	38	+10·0	24·84	3	0·48	
	Dec. 7	36	+17·2	25·13	3	0·12	
	8	38	−11·3	24·95	5	0·29	
1921	Jan. 13	36	+ 3·5	25·20	3	0·28	
	17	37	+12·3	24·96	3	0·18	*p* *f*
	Mar. 21	72	+14·9	25·04	7	0·40	
	Feb. 13	90	− 8·3	24·73	9	0·52	
	Mar. 8	37	+ 5·5	24·65	3	0·15	
	18	56	+19·7	25·23	7	0·30	
	22	65	+ 8·8	25·04	8	0·31	
	30	55	−16·2	25·38	4	0·43	
	Apr. 3	36	− 9·3	24·47	3	0·07	
	8	43	−10·5	24·16	3	0·30	
	19	45	+11·4	25·16	4	0·50	
	May 2	54	+12·2	25·19	5	0·42	
	9	37	+ 2·1	24·78	3	0·12	*p* *f*
	9	38	+ 1·7	25·17	5	0·05	
	June 1	53	+11·6	24·96	5	0·19	
	7	81	− 7·0	24·70	9	0·39	
	8	28	+14·2	25·17	2	0·05	
	25	65	+15·0	25·04	9	0·21	*p* *f*
	27	37	+14·1	25·33	5	0·10	
	July 12	63	− 2·6	24·69	7	0·25	
	25	37	− 9·5	25·24	3	0·04	

Date. Year.	First Measured on	Interval to Last Measure.	Mean Latitude.	Sidereal Rotation Period.	No. of Observations.	Extreme Range.	Remarks.
		Days.	°	Days.		Days.	
1921	Aug. 18	65	+12·0	25·18	9	0·90	
	22	37	+12·0	24·80	5	0·13	
	Sept. 1	55	−11·4	24·84	7	0·31	
	Oct. 17	65	+ 8·5	25·20	9	0·69	
	22	54	+ 5·2	25·16	8	0·47	
	Nov. 19	65	− 7·1	24·91	10	0·40	
	Dec. 9	63	+ 6·6	24·77	9	0·12	
	13	37	+12·4	24·75	5	0·32	
	28	37	− 7·2	24·52	6	0·54	
1922	Feb. 5	55	+ 9·0	25·01	6	0·27	
	Mar. 2	38	+ 8·8	25·18	6	0·23	
	7	55	+14·7	25·22	6	0·30	
	28	36	+11·9	24·90	5	0·07	*p*
	Apr. 23	38	+15·3	24·95	4	0·48	*f*
	1	81	+10·5	24·98	12	0·34	
	June 14	44	+10·0	24·57	4	0·10	
	July 18	54	− 6·4	24·97	5	0·24	
	Aug. 3	37	+12·2	24·68	3	0·10	
	27	46	−12·3	24·88	5	0·78	
	30	45	+ 5·7	24·93	4	0·50	
	Sept. 23	54	+13·4	25·33	3	0·28	
	Oct. 19	81	− 7·5	24·96	11	0·59	
	Nov. 13	38	+ 4·9	25·02	4	0·42	
	29	38	− 3·8	25·03	4	0·23	
1923	Aug. 30	38	−29·3	25·88	5	0·07	
	Sept. 26	38	−28·5	25·56	5	0·48	*p*
	27	31	−33·0	26·60	3	0·46	*f*

Note.—Faculæ marked *p* and *f* bracketed together represent the preceding and following parts of areas surrounding "bipolar" or "stream" type groups of sun-spots. In these cases the natural subdivision of the areas was very marked. The two faculæ marked * were seen in one transit only, but, as they were both small distinctive areas giving very consistent measures in longitude, the rotation periods derived from them were included.

Derivation of the Empirical Formula.—The separate values of the rotation period were then grouped according to their latitude in 5° zones and are given in the following table:—

Zone.	Mean Latitude of Faculæ $= \phi$.	Mean Sidereal Rotation Period.	Daily Mean Sidereal Motion $= \xi$.	No. of Faculæ.
	°	days	°	
0–5	3·1	24·88	14·47	14
5–10	7·2	24·87	14·47	40
10–15	12·6	24·97	14·42	41
15–20	17·6	25·19	14·29	30
20–25	22·7	25·44	14·15	31
25–30	27·3	25·76	13·97	26
30–35	33·9	26·34	13·67	9
35–40	37·1	26·84	13·41	5
45	45·4	28·06	12·83	1

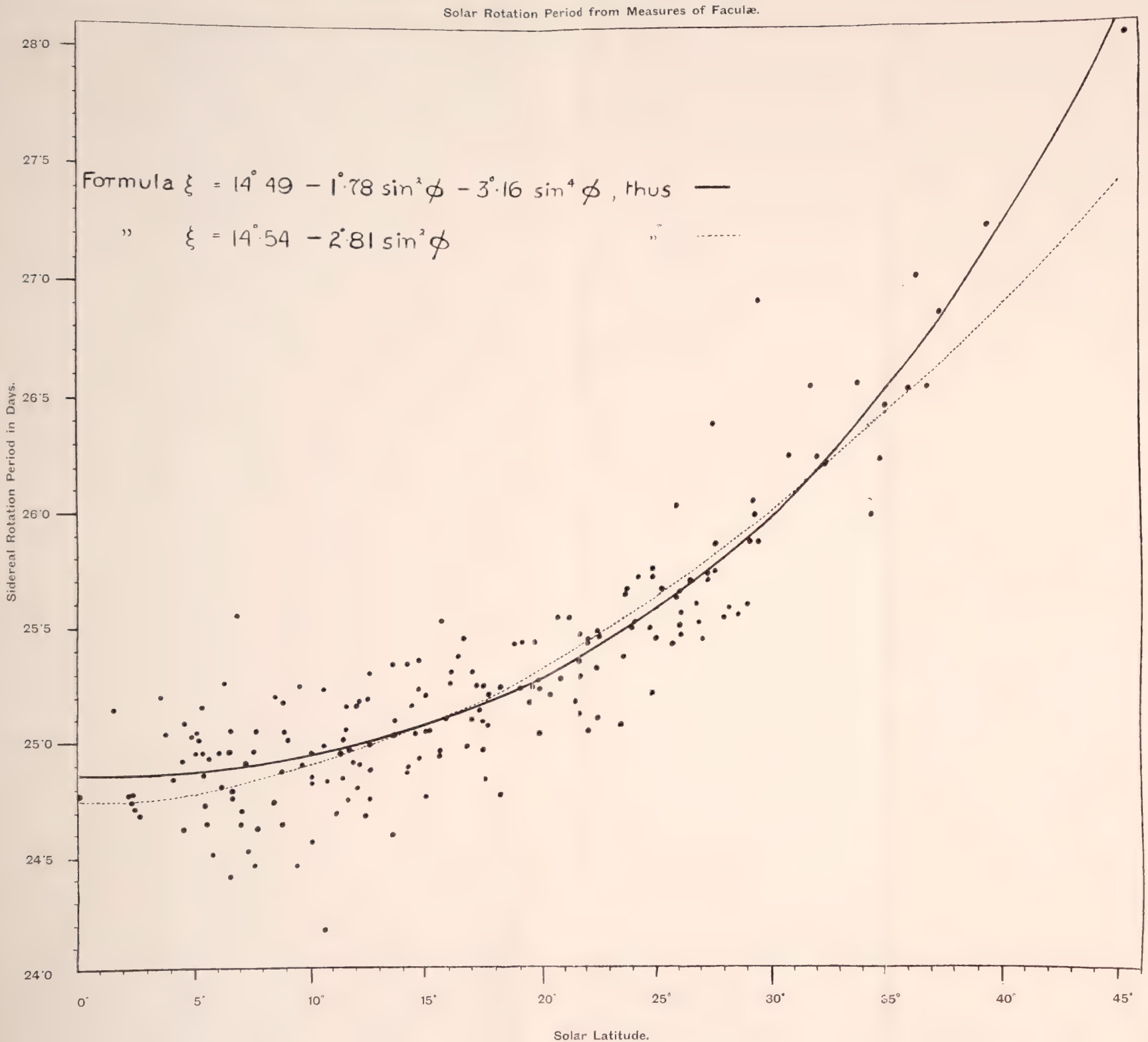
A solution of ξ in the form $a + b \sin^2 \phi$, by least squares, gave (1) $\xi = 14^\circ{\cdot}54 - 2^\circ{\cdot}81 \sin^2 \phi$, shown in the diagram by the dotted curve. The residuals showed a systematic run, indicating that the observations would be more closely satisfied by a higher power of $\sin \phi$. Accordingly, ξ was solved in the form $a + b \sin^2 \phi + c \sin^4 \phi$, and the result (2) $\xi = 14^\circ{\cdot}49 - 1^\circ{\cdot}78 \sin^2 \phi - 3^\circ{\cdot}16 \sin^4 \phi$ was obtained. This formula fits the observations very closely and is shown on the diagram by the thick line. The range of latitude for which ξ is known is, however, so small that b and c cannot be separated with any great accuracy.

A table of residuals between observed and computed values of ξ using both formulæ is given below :—

Mean Lat.	Observed Daily Sidereal Motion.	Computed from Formula I.	O – C.	Computed from Formula II.	O – C.	Weight.
°	°	°	°	°	°	
3·1	14·47	14·53	– ·06	14·48	– ·01	14
7·2	14·47	14·50	– ·03	14·46	+ ·01	40
12·6	14·42	14·41	+ ·01	14·40	+ ·02	41
17·6	14·29	14·28	+ ·01	14·30	– ·01	30
22·7	14·15	14·12	+ ·03	14·15	00	31
27·3	13·97	13·95	+ ·02	13·98	– ·01	26
33·9	13·67	13·67	00	13·63	+ ·04	9
37·1	13·41	13·52	– ·11	13·43	– ·02	5
45·4	12·83	13·12	– ·29	12·78	+ ·05	1

Throughout the computations each separate group of faculæ was given equal weight, although the respective rotation periods were not determined from an equal number of observations. This was considered to be the best procedure in order to avoid giving weight to any proper motion in longitude which a particular group might possess. Large proper motions are sometimes shown by the faculæ as well as sun-spots, and some of the outstanding points of the curve appear to be due to proper motion, and not to error of observation.

Discussion of Results.—The separate values of solar rotation were then tabulated in zones of every 5° in latitude in order to derive the


Solar Rotation Period from Measures of Faculæ.
Formula ξ = 14° 49 − 1°·78 sin²φ − 3°·16 sin⁴φ , thus
" ξ = 14°·54 − 2°·81 sin²φ "
Sidereal Rotation Period in Days.
28·0
27·5
27·0
26·5
26·0
25·5
25·0
24·5
24·0
0°
5°
10°
15°
20°
25°
30°
35°
40°
45°
Solar Latitude.

value for the mean deviation from the curve which is ±0·19 days. This range appears to be very satisfactory, considering the nature of the objects measured.

Fifty-three groups of faculæ were observed in the third rotation, and these were accordingly tabulated in order to see whether any systematic difference existed between the rotation periods derived from Rotations I.–II. and I.–III. The average difference obtained was ±0·13 days, and with respect to sign −0·03 days, where *minus* indicates I.–II. is the shorter-rotation period. The former value is of the same order as the mean deviation from the curve, and the very small negative residual is satisfactory in indicating that the groups of faculæ are fairly stable objects with regard to their centres of origin.

Finally, the results were divided so as to exhibit any possible differences in rotation periods existing between the northern and the southern hemispheres. The differences tabulated below are of the same order of magnitude as those found by other observers, and they tend in the same direction. With the present material available they can only be ascribed to accidental error.

Mean latitude . .		3°·1	7°·2	12°·6	17°·6	22°·7	27°·3	33°·9	37°·1
North	In period . .	+d·08	+d·12	+d·08	d·00	+d·09	−d·23	+d·06	−d·07
minus									
South	In diurnal motion	−°·05	−°·07	−°·05	°·00	−°·05	+°·12	−°·03	+°·04

Comparison with other Determinations.—A comparison at each 5° of solar latitude with the values derived from the most recent investigations is given in the table below:—

Faculæ (Greenwich) . . $\begin{cases} (1)\ \xi = 14^\circ\cdot54 - 2^\circ\cdot81 \sin^2\phi. \\ (2)\ \xi = 14^\circ\cdot49 - 1^\circ\cdot78 \sin^2\phi - 3^\circ\cdot16 \sin^4\phi. \end{cases}$

Sun-spots (recurrent), E. W. and A. S. D. Maunder $14^\circ\cdot43 - 2^\circ\cdot13 \sin^2\phi$.
Faculæ, Zô-Zè . . . . . $14^\circ\cdot47 - 2^\circ\cdot27 \sin^2\phi$.
Calcium Flocculi, Fox . . . . . $14^\circ\cdot56 - 2^\circ\cdot98 \sin^2\phi$.
Reversing Layer, Adams and Lasby . . . $14^\circ\cdot54 - 3^\circ\cdot50 \sin^2\phi$.

Daily Sidereal Motion.

Latitude.	Faculæ (Greenwich). Formula I.	Faculæ (Greenwich). Formula II.	Sun-spots.	Faculæ Zô-Zè.	Calcium Flocculi.	Reversing Layer.
°	°	°	°	°	°	°
0	14·54	14·49	14·43	14·47	14·56	14·54
5	14·52	14·48	14·41	14·46	14·54	14·51
10	14·46	14·43	14·37	14·41	14·47	14·43
15	14·35	14·36	14·29	14·32	14·36	14·31
20	14·21	14·24	14·18	14·21	14·21	14 13
25	14·04	14·07	14·05	14 07	14·03	13·91
30	13·84	13·85	13·90	13·91	13·81	13·67
35	13·62	13·56	13·73	...	13·58	13·39
40	13·38	13·22	...	...	13·33	13·10
45	13·13	12·81	...	...	13·07	12·79

The agreement is very close between the values derived from Formula I. and those of Prof. Fox's for the calcium flocculi.

In conclusion, the method employed in this paper appears to yield very satisfactory results from a comparatively small amount of material. It will be very desirable to collect data during the forthcoming cycle, especially with regard to the high latitude faculæ, to test the accuracy of both formulæ.

Royal Observatory, Greenwich:
1924 *March* 25.

A New Variable Star in Perseus. By A. Stanley Williams.

B.D.+40° 798 (9·1 mag.).

R.A. = 3h 28m 07s Decl. = +40° 16′·3 (1855).
R.A. = 3 31 06 Decl. = +40 25·5 (1900).

The variability of this star was first noticed by the writer about twenty years ago, when comparing two photographs taken with a 4·4-inch portrait lens, but, owing to ill-health and other causes, it was not until October 1921 that visual observations of it were commenced in order to confirm the variability and to ascertain the nature and period of variation. It turns out to be a rather interesting eclipsing variable of the β Lyræ type, or perhaps rather that of the Algol type. The observations also indicate the existence of unusual minor fluctuations in the form of the light-curve. It has, therefore, been thought advisable to give the actual observations in full.

Between 1921 October 4 and 1924 February 9, 258 observations were made on 92 nights. These are pretty well equally distributed among the three winter half-years 1921–22, 1922–23, 1923–24. The instrument used was a 6½-inch reflector, a power of 73 being always employed. Particulars of the comparison stars made use of and the deduced light-scale are given in Table I. It was found difficult to extend

TABLE I.

Star.	Name.	B.D. Mag.	Light-Scale.	Diff.	Adopted Mag.
			s		
a	B.D. 40°808	8·8	29·4		8·71
				6·0(3)	
c	B.D. 40 793	9·2	23·4		9·01
				1·8(172)	
e	B.D. 40 795	9·1	21·6		9·10
				7·2(191)	
f	B.D. 40 797	9·1	14·4		9·46
				5·2(190)	
g	B.D. 40 799	9·5	9·2		9·72
				9·2(67)	
b*	...	...	0·0		10·18

* This is a star, not in the B.D., a little following the variable.

the light-scale upwards owing to there being no suitable stars near, and in the absence of any photometric measures it is impossible to say if the adopted light-scale is exactly homogeneous throughout its extent or not, or what is the exact value in stellar magnitudes of a "step" in this scale. The magnitudes in the last column of the table are based on the assumptions that a step = 0·05 mag., which is probably nearly correct, and that the comparison star $e = 9{\cdot}1$ mag. But the magnitudes in the last column of the table are only added for the purpose of comparison, and in order to give some idea of the amount of the changes in brightness in terms of stellar magnitudes. In the present paper the observations of the brightness of the variable are reduced simply to steps of the light-scale.

The numbers in brackets following the differences in the fifth column of the table are the numbers of the comparisons upon which the difference depends. The comparison star a was only used for a few of the earlier observations. It should be remarked that the comparison stars generally are neither suitable nor conveniently placed for really satisfactory comparison with the variable, though they are the best that could be found, and that two of them, e and f, are not free from suspicion of being themselves slightly variable.

Table II. contains the individual observations. It is almost self-explanatory, but the numbers in column 4 require a few words. Previously to 1922 November 4, it was the writer's custom to note merely the transparency or quality of the sky, on a scale ranging from I. (very bad) to IV. (perfection). But subsequently to the above-mentioned date the observations have been weighted according to the degree of satisfaction given by them at the time, on a scale ranging from 1 (very bad) to 5 (perfect satisfaction). The Roman numerals I.–IV. in column 4 refer to the former mode of weighting, the Arabic figures 1–5 to the latter. Put shortly, the former give merely the state of the sky, whilst the latter are estimates of the actual value of the observations made at the time and, taking all things into consideration, including the state of the sky.

No attempt was made to reduce the observations or to discuss the results until late in the year 1923. The former should, therefore, be free from bias or pre-conceived impressions, excepting in the case of a few observations towards the end of the series. But there was an early idea that the variable must be some kind of eclipsing variable. This naturally resulted from the run of the observations about the times of principal minimum.

In the reduction of the comparisons in column 2, greater weight was given when the interval between the variable and the comparison star was small than when it was large, and usually double weight was assigned to a fractional determination, but no hard-and-fast rule was adopted, even if such were practicable. However, all the observations were finally reduced before any discussion was begun. The finally adopted brightness of the variable in steps is given in column 3 of the table. In no case have any of the figures been altered since the original reduction.

TABLE II.

Date. J.D. 242.	Observations.	Brightness. s.	Wt.	Phase. d.
2967·597	a10v15b, c3v	19·0	III.	1·832
2968·601	a16v18b, c0v	20·8	II.	2·836
2969·448	a12v, v12b, c0v	19·0	II.	3·683
2974·490	a15v16b, v3c	22·1	II.	1·565
·631	c4v10b	16·7	II.	1·706
2975·471	v2c, v15±b	25·4	III.	2·546
·512	v4c	27·4	III.	2·587
·575	c0v14b	23·4	III.	2·650
·662	c2v15b	20·6	II.	2·737
·696	c0v14b	23·4	III.	2·771
2977·412	c0v, v15b	23·4	IV.	4·487
·474	c0v	23·4	IV.	4·549
·564	c0v	23·4	IV.	4·639
·597	c0v	23·4	IV.	4·672
·653	c0v	23·4	IV.	4·728
2978·619	v1c	24·4	III.	5·694
2980·457	c0v	23·4	II.	0·373
·526	c0v	23·4	II.	0·442
2985·390	c10v12b	12·8	IV.	5·306
·396	c11v11b	11·7	IV.*	5·312
·404	c11v12b	12·2	IV.	5·320
·413	c12v13b	12·2	IV.	5·329
·417	c12v, e7v12b	12·9	III.	5·333
·429	c12v, e8v11b	12·1	II.	5·345
·485	c13±v, e9v9b	10·8	I.*	5·401
·520	c10v, e8v12b	13·1	II.	5·436
·540	c7v, e6v14b	15·5	III.	5·456
·566	c6v, e5v16b, v3f, v7g	16·6	II.	5·482
·583	c8v, e0v, v5f	18·8	III.	5·499
·622	c3v, e0v, v5f, v9g	19·9	III.	5·538
·648	c3v, e0v, v5f, v9g	19·9	III.	5·564
2986·387	v12g, v7f, e0v, c0v	21·9	III.	6·303
·412	v12g, v8f, e0v, c3v	21·4	III.	6·328
·547	v12g, v7f, e0v, c2v	21·4	IV.	6·463
·648	v11g, v6f, e2v, c3v	20·1	IV.	6·564
2987·371	f6v3g	10·9	IV.	0·127
·377	g1v, e13v, f8v10b	8·4	IV.	0·133
·389	g2v, f10v, e14v, v10b	7·3	IV.	0·145

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Brightness.	Wt.	Phase.
		s.		d.
2987·396	g2v, f10v, v10b	7·2	IV.	0·152
·419	g0v, f8v, e14v, c14v, v12b	8·8	IV.	0·175
·444	g2v, f8v, e14v, v12b	8·3	IV.	0·200
·487	v1g, f4v, e7v, c10v, v15b	12·3	IV.	0·243
·542	v7g, v2f, e3v, c8v	16·6	IV.	0·298
·585	v8g, v0f, e6v, c9v	15·4	IV.	0·341
·594	v10g, v5f, e4v, c7v	18·1	IV.	0·350
2991·339	v10g, v6f, e4v, c5v	18·9	III.	4·095
·379	v11g, v7f, e3v, c6v	19·4	III.	4·135
·475	v11g, v6f, e3v, c6v	19·2	III.	4·231
2992·396	v8g, v3f, e7v, c7v	16·4	II.	5·152
·450	v10g, v7f, e3v, c5v	19·4	III.	5·206
2993·377	v11g, v7f, e0v, c2v	21·1	III.	6·133
·439	v8g, v5f, e3v, c5v	18·4	III.	6·195
3001·362	g1v, f7v, e14v, v7b	7·7	IV.	6·958
·378	g1v, f8v, e14v, v8b	7·7	IV.	6·974
·419	g3v, f8v, e12v, v5b	6·4	IV.	7·015
·491	g3v, f7v, e12v, v5b	6·7	IV.	7·087
·529	g2v, f7v, e12v, v7b	7·7	IV.	7·125
·553	v2g, f5v, e12v, v6b	9·5	IV.	7·149
·591	v0g, f7v, e12v, v6b	8·3	IV.	0·028
3002·331	v10g, v7f, e0v, c3v	20·6	IV.	0·768
·387	v10g, v7f, v2e, c2v	21·4	IV.	0·824
·432	v12g, v8f, v0e, c3v	21·4	IV.	0·869
3005·344	v6g, v3f, e4v, c6v, v12b	15·9	IV.	3·781
·373	v7g, v4f, e2v	[18·1]	I.*	3·810
·433	v10g, v6f, e0v, c0v	21·1	III.	3·870
3035·235	v10g, v7f	[20·0]	I.*	5·033
3037·248	g0v, f7v, e10v, v8b	9·1	III.	7·046
·321	v2g, f6v, e10v, v8b	10·5	III.	7·119
·375	v0g, f8v, e10v, v8b	8·9	III.	0·014
·431	v0g, f7v, e10v, v8b	9·1	II.	0·070
3044·225	v7g, v3f, e4v, c7v	16·9	III.	6·864
·349	g6v, f10v, e12v, c12v, v7b	7·1	II.	6·988
3046·374	v7g, v4f, e4v, c4v	17·9	II.	1·853
·503	v10g, v6f, e5v, c8v	17·9	II.	1·982
3047·242	v8g, v5f, e3v, c5v, v14b	18·4	II.	2·721
·342	v12g, v8f, e3v, c5v	20·1	III.	2·821

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Brightness. s.	Wt.	Phase. d.
3047·492	v10g, v6f, e3v, c5v	19·1	II.	2·971
3049·235	v11g, v8f, e0v, c0v	22·0	III.	4·714
·343	v12g, v9f, e1v, c1v	21·8	III.	4·822
·419	v10g, v7f, e7v, c7v	17·9	III.	4·898
·501	v10g, v5f, e3v, c5v	18·9	III.	4·980
3053·256	v11g, v7f, e3v, c5v	19·6	III.	1·575
3055·441	v8g, v5f, e6v, c10v, v15b	16·1	II.	3·760
·463	v5g, v0f, e7v, c10v, v13b	14·0	III.	3·782
·506	v4g, v0f, e8v, c12v, v13b	13·3	III.	3·825
·552	v5g, f3v, e7v, c10v, v13b	13·3	III.	3·871
3058·347	v4g, v0f, e6v, c10v, v13b	14·0	II.	6·666
·437	v5g, v0f, e6v, c10v, v15b	14·5	II.	6·756
3059·356	v12g, v8f, e3v, c5v	[20·1]	I.*	0·516
3065·406	v10g, v5f, e0v, c2v	20·6	I.*	6·566
3072·330	v11g, v7f, v2e, c2v	21·6	II.	6·330
·401	v7g, v4f, e4v, c7v	17·1	II.	6·401
·503	v10g, v6f, e0v, c2v	20·8	II.	6·503
3076·369	v10g, v7f, e0v, c4v, v18b	20·6	II.	3·209
·415	v7g, v4f, e6v, c9v, v15b	15·9	II.	3·255
·514	v5g, v2f, e7v, c9v, v11b	14·1	II.	3·354
3080·335	g3v, f7v, e12v, v8b	7·5	II.	0·016
·364	g4v, f9v, e15v, v7b	5·9	II.	0·045
·459	g1v, f6v, e12v, v9b	8·7	I.	0·140
3081·381	v10g, v8f, e0v, c3v	20·9	II.	1·062
3090·367	v2g, f2v, e6v, c8v, v12b	13·3	III.	2·888
·409	v7g, v4f, e8v, c10v, v14b	15·3	III.	2·930
·477	v10g, v7f, e4v, c6v	18·9	III.	2·998
3104·375	v7g, v4f, e7v, c10v	15·6	II.	2·577
3107·358	v6g, v3f, e6v, c9v, v12b	14·9	III.	5·560
3112·361	v7g, v4f, e5v, c7v, v13b	16·1	III.	3·403
3334·478	v5g, f2v, e6v, c8v, v11b	13·7	IV.	3·571
·522	v8g, v4f, e4v, c7v	17·4	IV.	3·615
·594	v8g, v6f, e6v, c8v, v16b	16·9	IV.	3·687
·628	v10g, v7f, e2v, c5v	19·6	IV.	3·721
3335·505	v12g, v7f, [v7e], v2c	22·7	III.	4·598
·588	v10g, v6f, e0v, c1v	20·9	III.	4·681
·662	v12g, v7f, v1e, c1v	21·9	III.	4·755
3337·419	v12g, v8f, v2e, c0v	22·6	IV.	6·512

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Brightness. s.	Wt.	Phase. d.
3337·462	v12g, v7f, v0e, c2v	21·4	IV.	6·555
·546	v11g, v7f, v2e, c0v	22·1	IV.	6·639
·649	v10g, v5f, e2v, c5v	19·1	IV.	6·742
3352·709	v6g, v0f, e7v, c9v	14·6	IV.	0·323
3355·394	v10g, v5f, e3v, c6v	18·6	II.	3·008
·626	v8g, v3f, e5v, c8v	16·6	III.	3·240
3358·458	v10g, v7f, e1v, c7v	19·6	III.	6·072
·567	v12g, v7f, e3v, c5v	19·9	III.	6·181
3360·415	v10g, v6f, e2v, c4v	19·6	IV.	0·869
·569	v9g, v1f, e6v, c8v	16·0	IV.	1·023
·667	v5g, f3v, e9v, c12v, v10b	[11·9]	IV.	1·121
3363·467	v10g, v4f, e4v, c4v	[18·6]	1	3·921
3370·409	v7g, v0f, e5v, c7v	15·6	5	3·704
·459	v2g, f5v, e10v, c12v	11·0	5	3·754
·567	v1g, f6v, e9v, c12v, v10b	10·6	4	3·862
·594	v2g, f5v, e9v, c11v	11·4	5	3·889
·705	v7g, f0v, e5v, c8v	15·4	2	4·000
3371·382	v10g, v2f, e2v, c5v	18·3	2	4·677
3372·355	v6g, v2f, e4v, c7v	16·4	3	5·650
·528	v5g, v3f, e2v, c4v	17·6	3	5·823
·556	v8g, v6f, e5v, c5v	18·1	4	5·851
·628	v10g, v5f, e5v, c5v	18·4	2	5·923
3373·361	v12g, v7f, e0v, c2v	21·4	2	6·656
·413	v12g, v7f, e2v, c2v	20·9	2	6·708
3375·342	v6g, f2v, e6v, c7v	14·4	5	1·477
·409	v8g, f0v, e5v, c6v	16·0	5	1·544
·549	v7g, v2f, e2v, c3v	18·1	2	1·684
·601	v10g, v4f, e5v, c6v	17·9	3	1·736
3382·338	v11g, v7f, e2v, c2v	20·6	4	1·314
·383	v9g, v5f, c5v	18·1	3	1·359
3383·322	v6g, v0f, e6v, c7v	15·2	5	2·298
·546	v5g, f3v, e7v, c8v	13·9	3	2·522
·595	v10g, v3f, e5v, c5v	17·9	5	2·571
3384·645	v3g, f5v, e10v, c10v, v10b	11·3	3*	3·621
·664	v2g, f5v, e10v, c10v, v10b	11·1	2*	3·640
3386·608	v12g, v5f, e3v, c4v	19·6	2	5·584
·671	v11g, v7f, e5v, c7v	18·6	2	5·647
3387·342	v12g, v7f, e1v, c2v	21·1	4	6·318

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Brightness.	Wt.	Phase.
		s.		d.
3387·430	v12g, v8f, e0v, c1v	21·9	5	6·406
·549	v11g, v6f, e3v, c4v	19·6	4	6·525
3390·414	v12g, v5f, v2e, c2v	21·4	4	2·230
·501	v10g, v5f, v0e, c1v	21·1	5	2·317
3395·368	g0v, f6v, e12v, c14v, v7b	8·8	3	0·024
·469	g3v, f7v, e12v, c15v, v7b	7·7	1	0·125
·535	v1g, f5v, e12v, c14v, v8b	9·5	4	0·191
·594	v3g, f4v, e10v, c12v, v10b	11·1	3	0·250
3398·421	v11g, v5f, e5v, c7v	18·1	3	3·077
3399·459	v10g, v4f, e8v, c10v	16·1	3	4·115
3409·443	g3v, f7v, e12v, c14v, v9b	8·0	4	6·940
·494	g5v, f10v, e15v, c17v, v10b	6·3	4	6·991
·598	g1v, f7v, e12v, c14v, v8b	8·5	2	7·095
3412·319	v12g, v8f, e1v, c4v	20·8	5	2·656
·507	v10g, v6f, e4v, c7v	18·4	5	2·844
3413·235	v9g, v3f, e3v, c5v	18·1	3	3·572
3428·329	v10g, v5f, e3v, c5v	18·9	4	4·346
·381	v7g, v0f, e10v, c12v	13·6	4	4·398
3432·367	v8g, v2f, e8v, c10v	15·1	2	1·225
3455·343	v12g, v7f, v2e, c3v	21·6	5	2·722
·391	v12g, v6f, e3v, c5v	19·6	5	2·770
3470·371	v6g, v1f, e8v, c10v	14·4	3	3·431
·406	v8g, v0f, e12v, c14v	13·0	2	3·466
3492·340	v6g, v0f, e8v, c9v	14·4	3	3·921
3707·400	v6g, v0f, e8v, c10v	14·2	4	4·191
·632	v3g, f5v, e12v, c14v, v12b	10·6	4	4·423
·655	v0g, f8v, e10v, c14v, v12b	9·6	4	4·446
·694	v0g, f7v, e12v, c12v	9·4	4	4·485
3708·424	v11g, v6f, e6v, c8v	17·9	5	5·215
·580	v13g, v7f, e7v, c8v	18·4	4	5·371
3709·455	v5g, f1v, e8v, c10v, v12b	13·3	5	6·246
·480	v7g, f2v, e10v, c12v	11·9	5	6·271
·639	v10g, v4f, e5v, c7v	17·6	3	6·430
·686	v10g, v7f, e5v, c5v	18·9	5	6·477
3721·295	g2v, f5v, e9v, c11v, v11b	10·9	3	3·767
·385	g3v, f2v, e8v, c10v, v11b	11·3	4	3·857
3724·280	v5g, v0f, e7v, c9v, v13b	14·1	5	6·752
·381	v5g, v0f, e8v, c9v, v13b	13·9	5	6·853

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Brightness.	Wt.	Phase.
		s.		d.
3724·423	v5g, f2v, e10v, c12b, v12b	12·3	5	6·895
·530	v2g, f5v, e11v, c13v, v10b	10·3	5	7·002
3730·364	v7g, v3f, e3v, c4v	17·9	5	5·676
3733·263	v12g, v7f, e0v, c1v	21·6	5	1·416
·385	v12g, v5f, v1e, c1v	21·4	5	1·538
3734·287	v13g, v8f, e1v, c1v	21·9	5	2·440
·403	v12g, v6f, e1v, c1v	22·1	5	2·556
·534	v8g, v0f, e5v, c5v	16·2	5	2·687
·664	v6g, v0f, e6v, c8v	15·0	5	2·817
·688	v6g, f1v, e6v, c7v	14·8	4	2·841
3735·361	v8g, v3f, e7v, c7v	16·4	5	3·514
·399	v8g, v2f, e8v, c8v	15·6	5	3·552
·522	v10g, v5f, e5v, c5v	18·4	5	3·675
·648	v10g, v3f, e7v, c7v	16·9	4	3·801
3738·452	v9g, v3f, e5v, c5v	17·6	3	6·605
·486	v9g, v5f, e3v, c5v	18·6	4	6·639
·594	v10g, v5f, e4v, c6v	18·4	4	6·747
3747·331	v12g, v6f, e0v, c0v	21·6	3	1·164
3748·249	v11g, v5f, e0v, c1v	20·9	4	2·082
·364	v12g, v7f, e0v, c0v	21·9	4	2·197
·456	v11g, v4f, e2v, c3v	19·6	5	2·289
·551	v10g, v5f, e1v, c1v	20·4	5	2·384
·628	v11g, v5f, e1v, c2v	20·4	5	2·461
3749·682	v6g, v0f, e6v, c5v	15·9	3	3·515
3753·397	v1g, f4v, e12v, c13v, v7b	9·6	5	0·071
·421	g3v, f7v, e14v, c16v, v7b	7·0	2	0·095
·525	g2v, f7v, e13v, c14v, v8b	8·0	3	0·199
·580	g0v, f5v, e12v, c13v, v12b	10·0	3	0·254
·600	v2g, f2v, e11v, c12v, v11b	11·5	3	0·274
3758·262	v12g, v7f, e3v, c3v	20·4	5	4·936
3760·238	v4g, f2v, e5v, c7v, v10b	13·7	3	6·912
·247	v5g, f0v, e6v, c7v	15·0	4	6·921
3763·271	v12g, v4f, e0v, c3v	20·6	5	2·785
3767·255	v11g, v6f, e0v, c2v	20·9	5	6·769
·332	v9g, v5f, e3v, c5v	18·6	5	6·846
·374	v7g, v1f, e7v, c9v	15·2	5	6·888
·403	v6g, f1v, e8v, c10v	13·8	5	6·917
·448	g0v, f5v, e12v, c14v, v9b	9·3	5	6·962

TABLE II.—*continued.*

Date. J.D. 242.	Observations.	Bright-ness.	Wt.	Phase.
		s.		d.
3767·471	g4v, f8v, e14v, v8b	6·8	5	6·985
·510	g4v, f7v, e14v, v7b	6·8	5	7·024
·528	g2v, f6v, e12v, v8b	8·1	4	7·042
·576	g2v, f5v, e14v, c15v, v8b	8·0	4	7·090
·596	g3v, f7v, e14v, v9b	7·3	5	7·110
·608	g3v, f7v, e13v, v9b	7·5	5	7·122
·623	g2v, f7v, e14v, c16v, v9b	7·6	5	7·137
·631	gov, f5v, e14v, v10b	9·1	3	7·145
3770·519	v7g, fov, e6v, c7v	15·4	5	2·873
3810·319	v6g, fov, e5v, c7v, v14b	15·4	4	6·877
·357	gov, f5v, e10v, c12v, v12b	10·5	5	6·915
·464	g3v, f8v, v9b	6·9	2	7·022
3812·298	v10g, v5f, e3v, c5v	18·9	3	1·694
·446	v10g, v5f, e5v, c7v	17·9	3	1·842
·486	v12g, v7f, e7v, c7v	18·4	3	1·882
3816·282	v7g, vof, e7v, c10v	14·6	2	5·678
·289	v10g, v3f, e7v, c10v, v16b	16·4	2*	5·685
·444	v12g, v8f, e4v, c6v	19·6	4	5·840
3817·268	v8g, v1f, e8v, c10v, v13b	14·7	4	6·664
·374	v7g, v2f, e10v	[14·7]	1*	6·770
·378	v5g, vof, e10v, c12v, v12b	12·7	2	6·774
·405	v4g, f3v, e10v, c12v, v10b	11·5	3	6·801
·421	v3g, f4v, e11v, c14v, v10b	10·5	3	6·817
·437	v2g, f4v, e12v, c12v, v9b	10·5	3	6·833
·475	vog, f5v, e12v, c14v	9·4	3	6·871
·480	vog, f5v, e12v, c14v, v12b	9·8	3	6·876
3818·388	v10g, v5f, e7v, c10v	16·6	3	0·625
·419	v10g, v5f, e5v, c7v	17·9	3	0·656
3825·344	v9g, v5f, e7v, c10v	16·4	4	0·421
·376	v10g, v6f, e6v, c10v	17·1	4	0·453

Notes.—Observations marked * in column 4 were very probably affected by the presence of cloud, mist, fog, or haze. J.D. 3058·437 time not recorded, but probably about $10^h\ 30^m$. J.D. 3384, both observations *may* have been affected by haze or mist. J.D. 3721, variable apparently unsteady; it often seemed distinctly brighter than g. J.D. 3760, the two observations were made with special care, because a minimum was known to be approaching, so that they might be of use for fixing the time of this. J.D. 3817, none of the observations of this night free from suspicion of having been affected by haze.

The derivation of the elements of variation presented a little difficulty, as, owing to the long period of constant light at principal minimum, there are no observations of both the decrease and increase in brightness on the same night, and the change in brightness is at no time very rapid. After successive approximations, the finally adopted elements were:

$$\text{Principal minimum} = \text{J.D. } 2422987{\cdot}244 + 7^{d}\ 15965\ \text{E.}$$

Table III. gives the observed and computed times of 13 principal minima, with the differences O − C. It will be noticed that three of them are bracketed and given no weight. This is owing to the long

Table III.

E.	Observed Min.	Wt.	Computed Min.	O − C.
0	2422987·229	3	·244	−0·015
2	[3001·446]	0	·563	[− ·117]
7	[3037·349]	0	·362	[− ·013]
8	3044·555	1	·521	+ ·034
13	3080·261	1	·319	− ·058
57	3395·344	2	·344	·000
59	[3409·533]	0	·663	[− ·130]
103	3724·676	2	·688	− ·012
107	3753·362	2	·327	+ ·035
108	3760·549	1	·487	+ ·062
109	3767·682	3	·646	+ ·036
115	3810·600	2	·604	− ·004
116	3817·630	1	·764	− ·134

interval of constant light at minimum. There are in these three cases only observations during this period of constant light, and none during either the decrease or increase in brightness, so that they are of no use for improving the elements, though, of course, they confirm the general accuracy of the latter. The time of minimum, or rather the middle point of the period of constant light, derived from any of these three minima might well be in error to the extent of several hours. The average residual without regard to sign is $0^{d}{\cdot}039 = 56^{m}$, which is not excessive, having regard to the length of the period and other circumstances. The phase, reckoned from the previous principal minimum, according to the above elements for each individual observation, will be found in the last column of Table II.

The total number of observations is 258. Five of these were rejected in forming the mean light-curve, for the reason that they only had the lowest weight in Table II., and had almost certainly been affected by

cloud. One observation was rejected for discordance (J.D. 3360·667).* These 6 observations are bracketed in Table II. Also 8 observations on J.D. 2985 were formed into two groups, for the reason that at the time the rate ef change in brightness was quite unknown, and the observations were made at very short intervals, in case the change should turn out to be a very rapid one. These observations, all made at short intervals on the same night, would have had an undue swamping effect if used singly. Deducting the 6 rejected observations, and reckoning the above-mentioned 8 observations as 2, we have 246 remaining. These were formed into 31 mean groups or normals, 8 single observations to a normal, excepting the last two, where there are only 7 observations to a normal. In forming the means a reduced weight was given to a few of the single observations having a low weight in Table II. Some of the observations dated prior to J.D. 2985 were also only given half weight. In these early observations the comparison stars were few and unsuitable, and the brightness of the variable usually comes out too great.

TABLE IV.

Phase. d	Brightness. s	Phase. d	Brightness. s	Phase. d	Brightness. s
0·045	8·1	3·086	17·9	6·269	18·5
0·148	8·6	3·478	14·9	6·465	20·0
0·255	11·5	3·652	16·6	6·611	19·5
0·465	17·8	3·774	15·0	6·739	16·9
0·976	19·6	3·925	14·8	6·845	12·6
1·473	19·2	4·287	15·2	6·908	13·0
1·778	18·1	4·602	19·7	6·984	7·7
2·222	19·8	4·883	20·1	7·058	7·7
2·528	19·7	5·366	16·4	7·129	8·5
2·717	20·5	5·608	17·9		
2·838	17·3	5·904	19·3		

Table IV. gives the mean or normal values for the brightness of the variable, and these are also shown in diagram form in fig. 1. In this diagram a thick continuous line or mean curve has been carefully drawn through the dots representing the observations. A dotted line has also been inserted, ignoring the decline in brightness indicated about the middle points of the two maxima. It may be said to represent the light-curve of the star considered as a simple β Lyræ type or Algol type variable, the minor variations being here assumed to be due to errors of observation, though this last scarcely seems possible.

Considering the dotted curve first, it may be said to agree closely with that of an average β Lyræ type variable, excepting that the curves of the two maxima are rather flatter. It will be noticed that the actual

* This observation was made shortly before dawn of a November morning, when the observer, who had been observing all night, was probably tired. Having regard, too, to the time of year, it is likely that the air was saturated with moisture, and the small mirror of the telescope possibly slightly dewed.

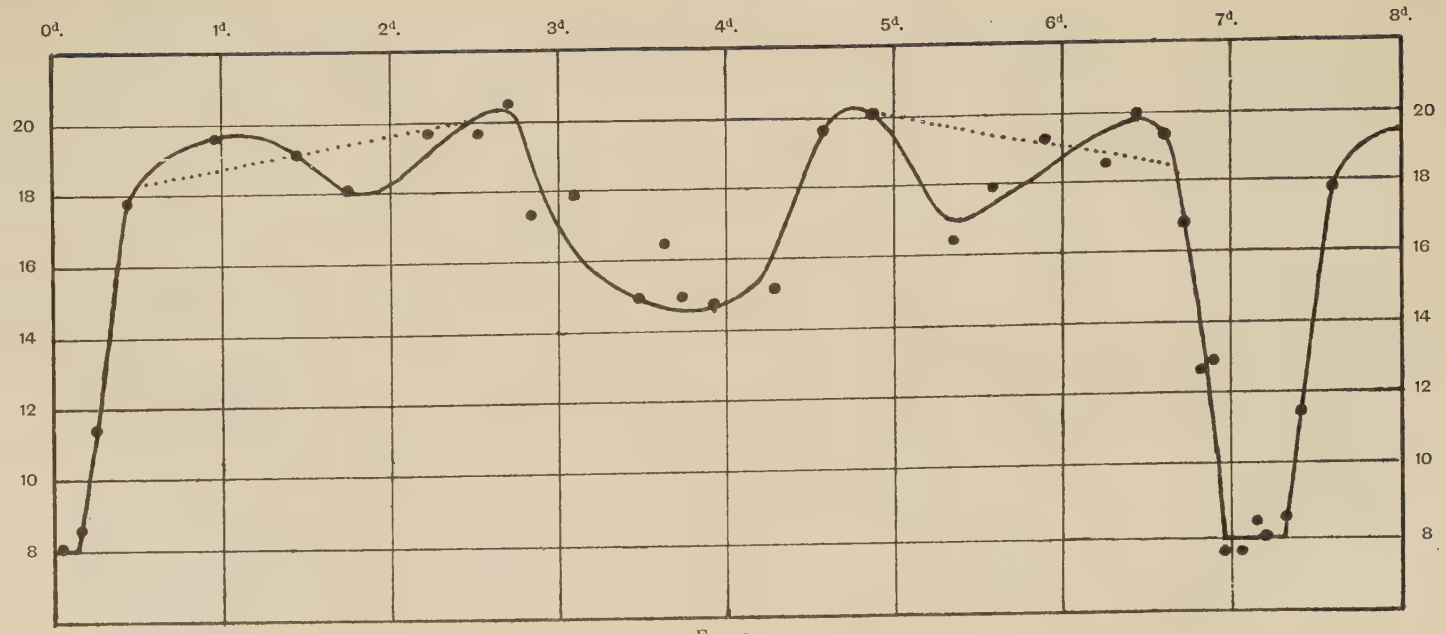


Fig. 1.

maximum points occur just before and after the secondary minimum, thus resembling Stebbins' light-curve of Algol, and perhaps due to a similar cause. In fact the dotted curve resembles the light curve of Algol more closely than it does that of β Lyræ. The secondary minimum appears to be a little eccentric with respect to the two adjacent principal minima, indicating probably a slight eccentricity of orbit. The principal minimum is remarkable for the long period of constant light, suggesting either a total or an annular eclipse. So long a period of constant light is unusual in the β Lyræ type of variable, though it is common enough in the case of an Algol type variable.

Coming now to the thick curve of fig. 1, it seems impossible to escape from the conclusion that there actually was a decline in brightness near the middle of each of the two maxima. The observations are so numerous and accordant, and made on so many different nights, that it seems impossible to ascribe these two drops in brightness simply to errors of observation. For the same reasons the suspected slight variability of two of the comparison stars could not produce the observed effect, whilst the fact of the comparison stars not being quite suitable could only slightly diminish the accuracy of the observations without detracting appreciably from the significance of the accordance.

Photometric observations of eclipsing variables are usually unfavourable to the reality of minor inflections in the form of the light-curve. The writer has not been able to refer to the light-curves of many stars of the kind, but the photographic light-curve of RX Herculis, published by Robert H. Baker and Edith E. Cummings in the *Laws Observatory Bulletin*, 25, 169, fig. 3, does show distinct irregularities somewhat similar to those here described, though on a smaller scale. In this curve there is a distinct depression near the middle of the second maximum, and a shallower and less definite one near the middle of the first maximum. The authors make the remark that "the inference is that the light of the system outside eclipse is variable, and that the fluctuations recur with considerable regularity at intervals of the eclipse period." These remarks would apply, perhaps, almost equally well to the present variable, though the changes observed in the latter might also be partly due to the effects of reflection, local differences in the surface brightnesses of the component stars of the system, and to their forms.

It should be remarked that the light-curve of RX Herculis above referred to is a photographic one, and that the writer's eyes are so insensitive to red light that, to a limited extent, his observations would resemble those made by photography. It seems desirable, therefore, that we should have not only a good photometric light-curve of the present variable, but a good photographic one also.

The scale of fig. 1 is too small, and the grouping of the observations too unfavourable to show clearly the long interval of constant light at principal minimum. Fig. 2 shows this on a larger scale. The means or normals are here formed of five observations in each case. It will be seen that the stationary period lasts for $0^d{\cdot}41$, or very nearly ten hours. The average deviation from the curve is here $\pm 0^s{\cdot}34 = 0{\cdot}017$ mag., so that it is doubtful if the slight departure from constancy suggested by

the observations has any meaning. The observation at $-^{d}\cdot36$ is very discordant. This is due to the fact that four out of the five observations forming the mean were made on the same night, J.D. 3817, and not one of the observations made on this night was quite free from suspicion

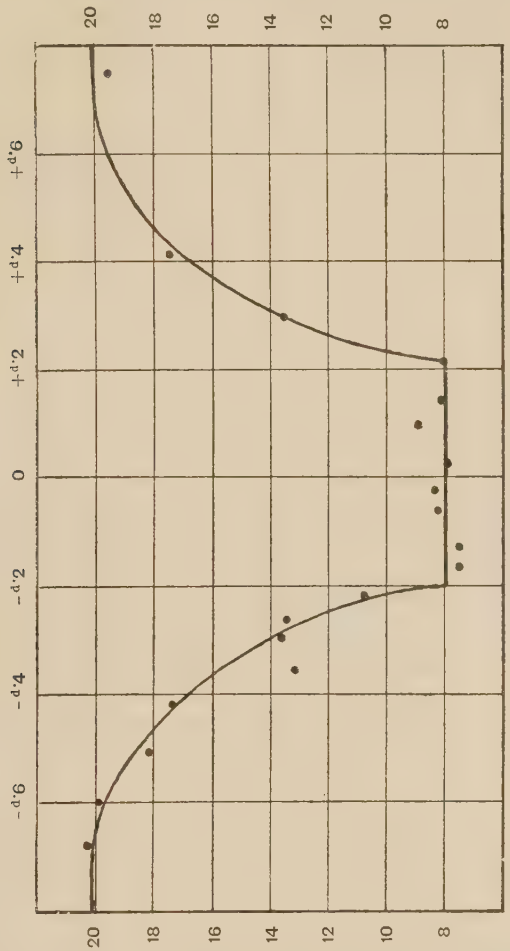


Fig. 2.

of having been affected by haze. The night was a very hazy one and, either directly or through the well-known Purkinje effect, the comparisons seem to have been all more or less prejudicially influenced.

Fig. 2 indicates that a small correction of $+0^{d}\cdot006$ is required to the mean epoch of principal minimum. Applying this, we have for the final elements of variation,

Principal minimum = J.D. $2422987\cdot250 + 7^{d}\cdot15965$ E.

Summary.

(1) The star B.D. 40° 798 is a variable of the β Lyræ or Algol type, with a pretty deep secondary minimum not exactly midway between the two adjacent principal minima.
(2) Period of variation . . . . . $7^{d}\cdot15965$
(3) Epoch of principal minimum . . . J.D. 2422987·250
(4) Brightness at principal minimum . . . . . 9·78 mag.
(5) Brightness at secondary minimum . . . . 9·43 ,,
(6) Brightness at the two maxima . . . . . 9·15 ,,
(7) Range . . . . . 0·63 ,,
(8) There seems to be a distinct decline in brightness, or subsidiary minimum, near the middle of each of the two maxima.

Polvarth, St. Mawes:
1924 *April* 2.

The Distribution of the Poles of 362 *Spiral Nebulæ.*

By C. C. L. Gregory, M.A., D.I.C. (Plates 11, 12, 13.)

1. *Introductory.*—It has been shown by Mr. J. H. Reynolds that the inclinations of the planes of spiral nebulæ to the line of sight do not follow a random distribution, there being a deficit of large inclinations among the spirals of large angular size. This deficit gradually disappears with decreasing angular dimensions, until for spirals of less than 2′ there are an approximately equal number having inclinations over and under 30°, this being the inclination which divides a hemisphere into two equal areas.

Since the nebulæ considered are by no means evenly distributed over the celestial sphere, this arrangement would necessitate a systematic grouping of the poles of the nebulæ, the poles being defined as the diametrically opposite points at which the celestial sphere is intersected by the axes of symmetry of the nebulæ.

If we suppose a nebula to be a flat circular disc, seen in projection as an ellipse, then the co-ordinates of the poles are known if the ratio of the apparent axes and the position angle of one of them be given.

This paper is an attempt to investigate whether or not there is a systematic distribution of the poles of spirals, use being made of such data as is available.

2. *Discussion of Data.*—For the north equatorial hemisphere 188 spirals and spindles were selected out of a total of 260 of 1′ and greater given in *Publications of the Lick Observatory*, 13, and for the southern hemisphere 140 out of 303 in the above and in the *Helwan Observatory Bulletins*, Nos. 9, 15, 21, and 22; also 34 south circumpolars measured by Mr. J. H. Reynolds from the Franklin-Adams plates.

These selected nebulæ were then divided into seven groups as follows:—

Group U_n comprised all the nebulæ of north declination for which a, b, and Ω were given (a and b being the greatest and least angular dimensions and Ω the position angle of the direction of greatest elongation measured from 0° to 180°).

Group V_n – spindles and edgewise spirals for which $b = 0$.

Group W_n – nearly round nebulæ (excluding globular or spheroidal nebulæ which were not included at all).

U_s, V_s, and W_s – correspondingly for the southern hemisphere from declination 0° to −45°, which is the zone covered by the Helwan photographs.

X_s – all available nebulæ from Franklin-Adams plates in the zone −45° to −90°.

Where possible, comparison was made between the dimensions given and photographs, and a few nebulæ were rejected in consequence of obvious non-agreement. No nebulæ were, however, added to or rejected from the first selection. The right ascensions and declinations (α, δ) of the poles were then computed from the following equations:—

$$\begin{aligned} \cot(\alpha - \alpha_0) &= \sin\delta_0 \tan\Omega \pm \cos\delta_0 \cot i \sec\Omega, \\ \sin\delta &= \sin\delta_0 \cos i \mp \cos\delta_0 \sin i \sin\Omega, \\ \sin(\alpha - \alpha_0) &= \pm \sin i \cos\Omega \sec\delta, \end{aligned}$$

where (α_0, δ_0) are the co-ordinates of the nebula, and $i = \cos^{-1}(b/a)$. These give for each nebula two pairs of diametrically opposite poles, only one pair being real, the other being spurious. This ambiguity cannot be avoided unless it is known which end of the apparent minor axis is nearer to the observer. This might become known in cases where absorption is present, but no attempt has been made to remove this ambiguity. The four poles lie on a great circle which is the continuation of the minor axis, and become a single pair when either $a = b$ or $b = 0$. As only two poles can be in one hemisphere (the others being diametrically opposite), use was made of the stereographic projection of a single hemisphere, the centre of projection being one celestial pole and the map centre the other. This projection has the advantage of conformality, but it does not represent by equal areas on the map equal areas of the sphere. Consequently, in order to estimate numerically the distribution of the poles, each map was divided into eighteen equal areas bounded by the equator, two circles of declination 19° 28′ and 41° 48′, and by six equally spaced meridians. These areas are designated by the letters A, B, C, etc., for the northern hemisphere, while A′, B′, C′, etc., denote the areas diametrically opposite on the southern hemisphere. Unfortunately for statistical purposes the data is necessarily of a highly selective nature, and we must consequently see in what direction this will influence the distribution of the poles. A comparison of various independent measurements of Ω for the same nebula shows this to be reliable to within 10° or less, except in the case of the comparatively few nebulæ measured, for which i is less than about 45°. Moreover, the *selection* in this case is independent of Ω and depends only on the value of i, since the position-angle is more

readily and accurately determined where i is large. With a supposed random distribution of nebular planes, the effect of this will be to cause an apparent uniform clustering of the poles of a group of nebulæ around a great circle whose pole is the centre of the group. This is illustrated by the group of 14 nebulæ shown in fig. 1 at R.A. 182° Dec. + 14°. Their poles are shown in fig. 2, where the centres of the lines joining the two possible poles are marked with a X. The distribution of the crosses would be consistent with either a random distribution of planes combined with this principle of selection, or with the supposition that the planes tend to conform to a plane whose pole is at the centre of the group considered. Now it is possible to test which hypothesis is the correct one by adding to fig. 2 all the poles of the nearly round nebulæ in duplicate, and if the pole density near the group considered is then less than the pole density 90° away, then the second hypothesis may be considered to be the correct one. This, in effect, is what has been done by combining the poles of all the groups. Undue weight is given to the nearly round nebulæ, because these are not subject to any selection. The result is, I think, a very striking one. There is a remarkably uniform distribution of poles over the sphere with the exception of the diametrically opposite areas K and K′, when the density falls away to almost zero. These two areas are exceptional in that they contain respectively the north and south galactic poles, and in no other respect.

We are now in a position to test whether or not the distribution actually found is one that could be expected to arise from chance alone. If we take the total number of poles in one hemisphere, as in Table II., one-half of them are spurious.

If E_1 be the average error and $E_1(K)$ the error from the mean for the area K, then

$$E_1 = 6{\cdot}44 \quad \text{and} \quad E_1(K)/E_1 = 4{\cdot}04.$$

In Table II., column 24, the spurious poles of groups U_n and U_s are supposed to be uncorrelated as to position with the real poles, and the necessary subtraction has been made. Columns 17 to 23 are obtained from columns 9 to 15 by treating these columns as indicated in the table.

In this case $E_2 = 4{\cdot}53$ and $E_2(K)/E_2 = 4{\cdot}22$. In both cases the next largest corresponding ratio occurs in the adjacent area R.

The slight excess of poles in D is explained by the aggregation of a large number of small nebulæ in this region, among which the grouping of the poles is less marked, as is shown in Table III.

Table IV. shows the distribution of poles according to the type of the nebulæ in groups U_n and U_s.

3. *Description of Maps.*—Fig. 1 shows the distribution of the nebulæ of group U_n, the attached numbers referring to the group number of the nebula in the first column of Table I. Fig. 1 should be compared with fig. 2, which represents the distribution in the northern hemisphere of both the possible poles of the nebulæ in group U_n. In order to facilitate identification the poles are joined by a straight line. In cases where the nebula lies between its poles, lines are drawn from

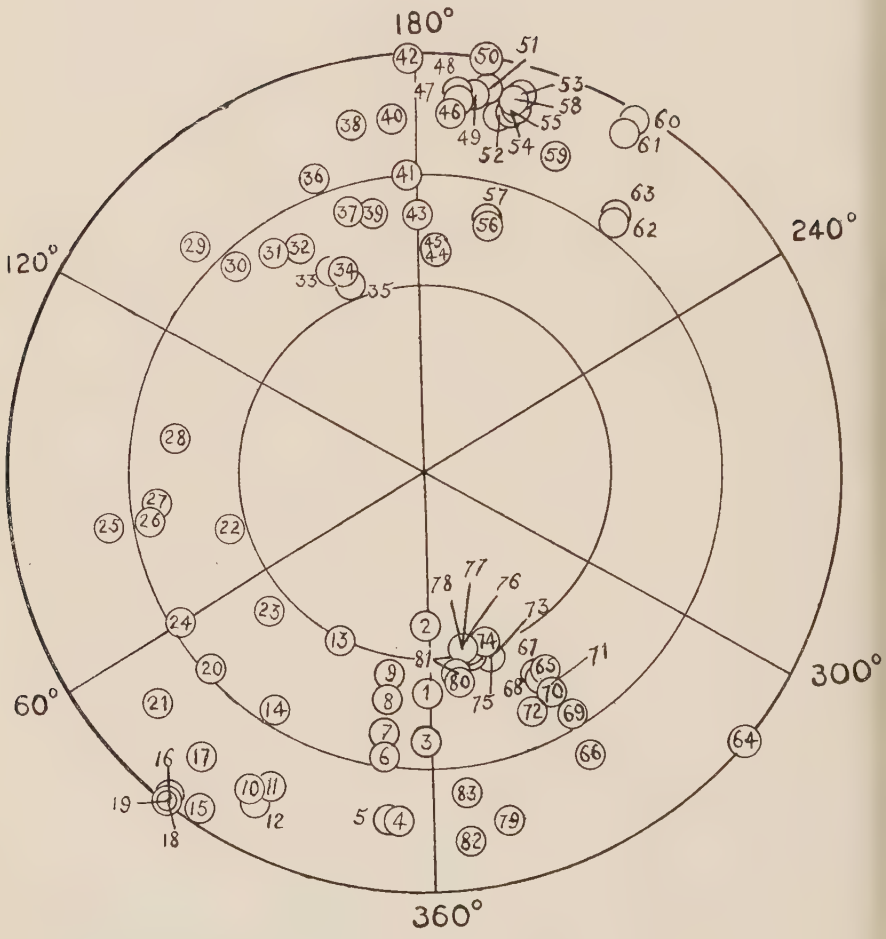


Fig. 3.—Group U_8 (Nebulæ).

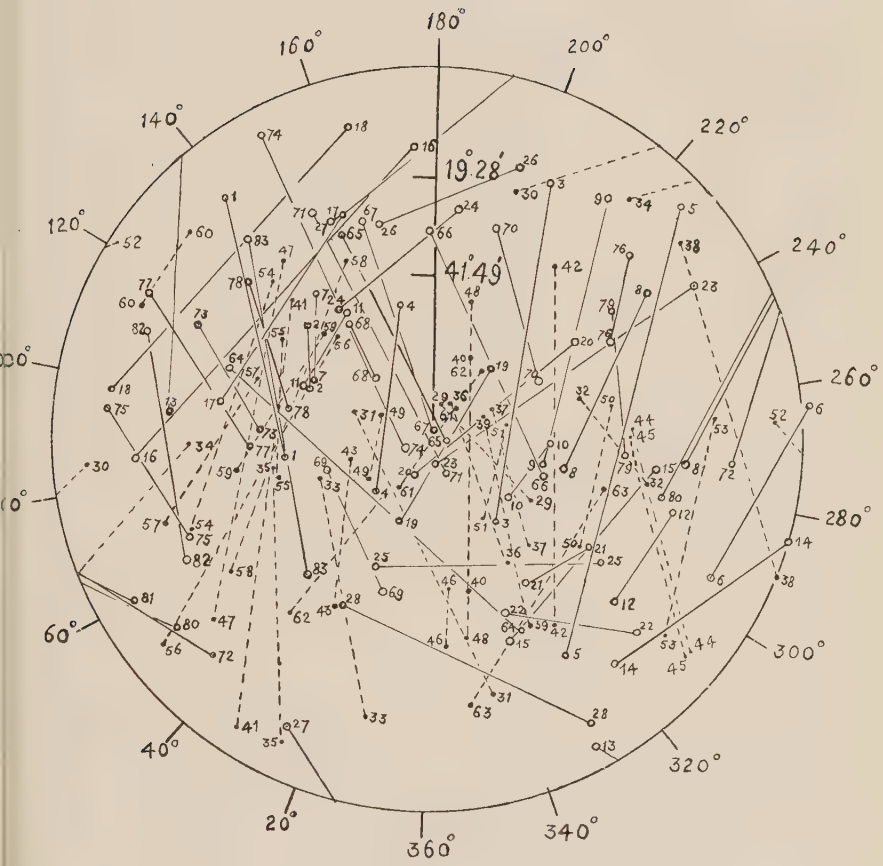


Fig. 4.—Group U_8 (Poles).

NEBULÆ (362).

NORTH.

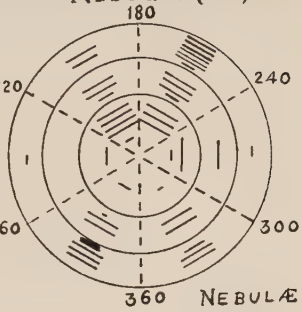


Fig.5

POLES (364).

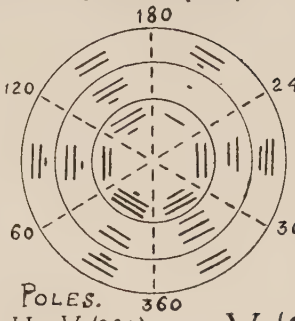


Fig.6

NEBULÆ $1' \& > 1'$

NORTH 260
SOUTH (303).

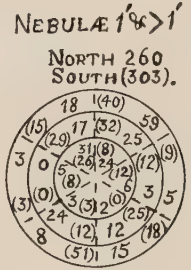


Fig.11

NEBULÆ
$U_n \& V_n$ (140).

Fig.8

POLES.
$U_n \& V_n$ (280).

Fig.9

W_n (48).

Fig.10

NORTH.

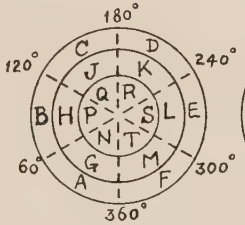


Fig.7

SOUTH.

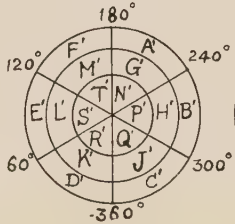


Fig.12

$U_s \& V_s$ (109).

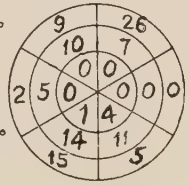


Fig.13

$U_s \& V_s$ (218).

Fig.14

W_s (31).

Fig.15

POLES.
X_s (68).

Fig.16

TABLE I.

Group U_n.

Group No.	N.G.C.	α_1.	δ_1. +	α_2.	δ_2. +	Group No.	N.G.C.	α_1.	δ_1. +	α_2.	δ_2. +
1	29	118°	5°	84°	27°	38	4312	289°	6°	104°	13°
2	91	184	50	184	86	56	4388	5	69	5	85
3	697	142	21	112	32	57	4389	355	18	340	69
4	740	152	25	131	40	58	4402	10	65	31	83
5	772	181	28	77	58	59	4414	240	1	114	31
6	818	195	30	174	67	60	4429	23	56	137	77
7	972	158	12	121	32	61	4448	6	41	6	81
8	1055	174	73	84	73	62	4488	299	26	262	31
9	2768	132	5	306	64	63	4501	320	24	250	42
10	2841	260	4	241	29	64	4527	56	64	127	68
11	II. 2487	88	0	202	24	65	4536	309	54	254	56
12	2903	22	6	70	24	66	4535	62	20	125	30
13	2976	198	7	30	30	67	4559	316	28	287	45
14	3079	254	1	246	13	68	4562	61	31	91	46
15	3169	20	45	97	50	69	4567	46	36	132	54
16	3198	20	10	48	41	70	4568	338	53	250	71
17	3227	276	13	228	30	71	4631	15	51	18	63
18	3254	27	28	55	46	72	4654	328	39	258	56
19	3489	12	48	103	69	73	4666	87	44	113	44
20	3556	352	23	354	45	74	4712	127	2	263	19
21	3623	274	4	245	11	75	4725	220	1	151	43
22	3627	241	4	97	4	76	4747	78	22	106	31
23	3675	246	13	94	13	77	4736	358	1	264	75
24	4036	6	13	8	42	78	4826	352	34	255	70
25	4051	332	9	290	61	79	5005	40	30	66	62
26	4062	354	38	338	77	80	5033	134	14	265	21
27	4096	70	5	82	19	81	5055	11	19	352	74
28	4100	293	0	274	23	82	5194	251	5	114	41
29	4145	181	6	1	86	83	5248	330	30	266	40
30	4192	296	24	264	32	84	5377	79	16	95	35
31	4206	82	3	102	7	85	5395	274	18	142	18
32	4212	31	40	132	62	86	5545	58	38	172	60
33	4216	81	15	97	19	87	5560	13	69	247	76
34	4220	315	20	303	36	88	5566	92	23	154	28
35	4244	54	34	60	40	89	5673	355	10	331	44
36	4258	117	4	271	23	39	5678	290	17	136	22
37	4274	357	43	346	75	40	5676	77	0	116	49

TABLE I.—*continued.*

Group U_n—*continued.*

Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.	Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.
			+		+				+		+
41	5740	0°	8°	260°	11°	55	7332	86°	16°	68°	24°
42	5907	341	9	336	17	90	7339	149	55	180	77
43	5981	7	16	3	25	91	7448	94	3	49	15
44	5985	307	23	157	28	92	7463	152	66	113	82
45	6070	99	46	202	48	93	7497	232	40	266	44
46	6181	318	7	176	7	94	7537	195	73	304	81
47	6207	133	2	166	26	95	7541	146	71	25	78
48	6503	54	4	48	28	96	7601	168	28	348	44
49	6643	322	12	152	31	97	7640	88	2	79	10
50	6690	202	8	14	15	98	7780	57	4	297	4
51	6928	89	58	20	70	99	7782	117	1	57	8
52	6930	206	8	224	11	100	7817	241	36	264	45
53	7217	156	6	160	68	101	224	245	15	266	35
54	7331	86	5	67	19	102	3031	92	29	243	44

Group U_s.

Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.	Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.
			−		−				−		−
1	10	82°	48°	139°	10°	20	1300	26°	79°	234°	40°
2	24	112	52	132	42	21	1337	325	41	302	37
3	45	319	59	204	16	22	1808	336	37	312	16
4	151	45	68	166	50	23	1326	350	82	238	11
5	191	327	21	226	5	24	1518	144	44	186	27
6	247	294	10	83	1	25	1954	22	50	303	32
7	253	116	51	141	38	26	1964	165	29	197	16
8	254	280	50	234	20	27	2090	155	24	189	9
9	300	280	56	215	12	28	2280	28	38	329	8
10	(127)	304	61	270	55	29	2781	198	79	299	55
11	755	112	51	145	45	30	2835	198	20	84	5
12	779	310	23	285	22	31	3054	110	66	345	20
13	808	94	20	330	4	32	3175	252	46	280	29
14	908	319	14	284	0	33	3281	70	56	12	16
15	936	338	30	276	28	34	3347	88	25	218	10
16	1068	85	14	175	14	35	3358	79	44	25	6
17	1084	110	31	158	24	36	3456	206	78	327	48
18	1087	98	9	163	8	37	3511	235	69	316	48
19	1090	20	64	219	62	38	3672	230	9	290	0

TABLE I.—*continued.*

Group U_s—*continued.*

Group No.	N.G.C.	α_1	δ_1.	α_2.	δ_2.	Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.
			−		−				−		−
39	3673	243°	723°	30°	31°	62	5716	219°	62°	39°	29°
40	3905	205	62	344	45	63	5728	284	38	350	19
41	4027	134	34	33	4	64	6962	333	31	109	31
42	4030	214	30	325	29	65	7163	270	85	155	28
43	(2995)	77	66	28	38	66	7171	287	55	179	32
44	(3253)	266	33	308	6	67	7172	230	86	161	28
46	4487	351	47	354	32	68	7174	139	66	143	49
47	4504	140	27	49	17	69	7184	71	58	17	44
48	4520	196	48	349	34	70	(5168)	240	53	196	29
49	4602	118	74	55	69	71	7214	331	79	151	21
50	4642	258	38	303	38	72	7314	43	12	273	12
51	4697	258	66	322	61	73	7410	92	42	115	21
52	4818	266	4	121	1	74	7412	73	82	150	4
53	4941	264	16	308	12	75	7456	68	21	94	9
54	4942	134	29	69	22	76	7582	240	32	226	18
55	4948	124	40	76	44	77	7590	87	39	116	10
56	4965	138	49	51	7	78	7599	111	49	130	24
57	4980	109	39	71	18	79	7606	272	34	234	28
58	4981	153	34	55	25	80	(5332)	282	25	52	10
59	5254	131	46	80	36	81	7713	273	21	60	6
60	5496	130	10	114	10	82	7721	62	19	110	13
61	5506	212	79	32	73	83	7723	41	39	137	18

Group V_n.

Group No.	N.G.C.	α	δ	Group No.	N.G.C.	α	δ
			+				+
1	678	232°	66°	20	4490	335°	44°
2	891	287	18	21	..	94	42
3	1023	237	50	22	4517	7	84
4	1560	316	6	23	4526	290	55
5	2146	213	6	24	4533	278	20
6	II. 2233	20	12	25	4565	302	40
7	2683	17	32	26	4605	340	25
8	3628	321	73	27	4660	356	77
9	..	88	0	28	4710	93	27
10	..	293	17	29	5003	85	22
11	4013	26	42	30	5529	355	46
12	4026	271	2	31	I. 1029	329	18
13	..	284	12	32	5746	310	10
14	..	296	56	33	5859	338	46
15	4222	67	57	34	5866	6	27
16	4252	89	33	35	n.f. 6482	335	36
17	4248	341	39	36	n.f. 7457	97	34
18	4270	295	18	37	7457	110	46
19	4301	281	55	38	7814	106	46

TABLE I.—*continued.*

Group V_s.

Group No.	N.G.C.	α	δ	Group No.	N.G.C.	α.	δ.
			−				−
1	7	258°	22°	15	4594	359°	79°
2	148	187	58	17	4674	316	78
3	586	113	10	18	4672	70	29
4	625	203	48	19	4703	285	25
5	955	305	20	20	4757	271	46
6	1381	172	35	21	4784	315	72
7	(346)	262	69	22	4856	93	39
8	1421	324	0	23	4958	104	10
9	1888	172	34	24	..	304	25
10	2811	42	19	25	5170	301	49
11	3113	52	44	26	(5078)	151	72
12	3333	268	24	27	7180	202	58
13	3404	21	76	28	7300	74	24
14	3936	28	60				

Group W_n.

Group No.	N.G.C.	α.	δ.	Group No.	N.G.C.	α.	δ.
			+				+
1	185	8°	48°	25	3938	177°	45°
2	198	8	2	26	4041	179	63
3	278	12	47	27	4254	184	15
4	488	19	5	28	4303	184	5
5	514	20	12	29	4321	184	16
6	628	23	15	30	4340	184	17
7	..	28	25	31	4394	185	19
8	1642	70	0	32	4625	189	42
9	2507	99	16	33	4639	190	14
10	2532	121	34	34	4647	190	12
11	2649	130	35	35	5457	210	55
12	2655	131	79	36	5569	214	4
13	2775	136	7	37	5829	225	24
14	3147	152	74	38	5850	225	2
15	3184	153	42	39	5957	233	12
16	3344	160	25	40	6412	263	76
17	3351	160	12	41	6654	276	73
18	3367	160	14	42	6946	308	60
19	3368	160	12	43	6951	309	66
20	3619	168	58	44	7156	328	3
21	3681	170	17	45	7177	330	17
22	3684	170	18	46	7479	345	12
23	3686	170	18	47	7753	356	29
24	3892	176	49	48	7779	357	7

TABLE I.—*continued.*

Group W_s.

Group No.	N.G.C.	α.	δ.	Group No.	N.G.C.	α.	δ.
			−				−
1	157	8°	9°	17	4995	196°	7°
2	165	8	11	18	5236	203	29
3	255	11	12	19	5247	203	17
4	941	36	2	20	5426	210	6
5	945	36	11	21	5427	210	6
6	948	36	11	22	5494	212	30
7	1309	50	16	23	5968	234	30
8	(343)	52	19	24	6814	294	11
9	2217	94	27	25	7371	340	12
10	3124	150	10	26	7418	343	38
11	3450	161	20	27	7421	343	38
12	3513	165	23	28	7424	343	42
13	3887	176	16	29	7552	348	43
14	4038	179	18	30	7723	354	14
15	4304	184	33	31	7755	358	31
16	4902	194	14				

Group X_s.

Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.	Group No.	N.G.C.	α_1.	δ_1.	α_2.	δ_2.
			−		−				−		−
1	406	119°	37°	318°	20°	18	6943	173°	21°	169°	5°
2	1249	188	62	210	4	19	7038	184	66	154	10
3	1313	130	29	336	26	20	7059	187	54	346	4
4	1448	163	33	179	15	21	7064	147	48	145	25
5	1511	281	47	87	12	22	7083	45	31	255	23
6	1515	151	23	352	6	23	7090	188	32	182	20
7	1527	214	70	232	11	24	7124	205	46	183	11
8	1553	309	45	284	2	25	7125	138	59	322	1
9	1558	244	64	244	26	26	7179	142	44	154	6
10	1602	226	64	237	4	27	7307	157	63	157	35
11	1617	227	64	238	4	28	7368	231	50	200	20
12	2442	230	41	70	16	29	7462	171	68	157	29
13	4835	87	42	60	11	30	7582	168	69	168	25
14	6300	134	48	291	8	31	II. 1970	193	60	212	24
15	6707	144	44	125	9	32	,, 4720	187	36	334	16
16	6887	122	13	122	61	33	,, 5052	175	24	171	6
17	6909	36	28	242	1	34	,, 5152	149	68	329	8

Table II.

	Nebulæ.							
	(1) U_n.	(2) V_n.	(3) W_n.	(4) U_s.	(5) V_s.	(6) W_s.	(7) X_s.	(8) Total.
A	2	0	4	16	10	5	0	37
B	0	0	2	0	0	1	0	3
C	4	1	7	5	0	2	0	19
D	21	12	10	11	4	8	0	66
E	1	0	0	1	1	0	0	3
F	10	1	4	6	3	3	0	27
G	7	2	1	4	3	4	0	21
H	0	0	0	0	0	0	0	0
J	4	1	3	11	0	4	2	25
K	17	3	1	11	3	0	0	35
L	2	1	0	5	0	1	0	9
M	6	2	1	8	2	1	0	20
N	0	1	2	0	0	0	1	4
P	0	2	0	0	0	0	3	5
Q	9	6	7	4	0	2	15	43
R	16	6	2	0	1	0	5	30
S	3	0	2	0	0	0	8	13
T	0	0	2	0	0	0	0	2
	102	38	48	82	27	31	34	362

	Poles.							
	(9) U_n.	(10) V_n.	(11) W_n.	(12) U_s.	(13) V_s.	(14) W_s.	(15) X_s.	(16) Total.
A	11	2	8	9	0	10	5	45
B	21	2	4	6	0	2	3	38
C	9	0	14	7	2	4	5	41
D	6	2	20	7	2	16	0	53
E	17	10	0	11	4	0	3	45
F	8	6	8	7	0	6	7	42
G	11	4	2	7	0	8	6	38
H	14	8	0	10	6	0	2	40
J	10	0	6	14	4	8	2	44
K	5	0	2	5	0	0	2	14
L	11	4	0	10	6	2	2	35
M	10	6	2	14	4	2	9	47
N	17	4	4	9	6	0	10	50
P	12	8	0	10	4	0	2	36
Q	12	0	14	9	8	4	1	48
R	5	4	4	7	6	0	0	26
S	13	6	4	14	0	0	0	37
T	12	10	4	8	2	0	9	45
	204	76	96	164	54	62	68	724

	Poles.							
	(17) (9)−6.	(18) ½(10).	(19) ½(11).	(20) (12)−4.	(21) ½(13).	(22) ½(14).	(23) (15)−2.	(24) Total.
A	5	1	4	5	0	5	3	23
B	15	1	2	2	0	1	1	22
C	3	0	7	3	1	2	3	19
D	0	1	10	3	1	8	−2	21
E	11	5	0	7	2	0	1	26
F	2	3	4	3	0	3	5	20
G	5	2	1	3	0	4	4	19
H	8	4	0	6	3	0	0	21
J	4	0	3	10	2	4	0	23
K	−1	0	1	1	0	0	0	1
L	5	2	0	6	3	1	0	17
M	4	3	1	10	2	1	7	28
N	11	2	2	5	3	0	8	31
P	6	4	0	6	2	0	0	18
Q	6	0	7	5	4	2	−1	23
R	−1	2	2	3	3	0	−2	7
S	7	3	2	10	0	0	−2	20
T	6	5	2	4	1	0	7	25
	96	38	48	92	27	31	32	364

TABLE III.

Distribution of Total Poles, with regard to Size of the Nebulæ.

Position Letter.	<1′·4	1′·4 to 2′·3	2′·3 to 3′·6	3′·6 to 5′·1	5′·1 to 7′·0	7′·0 and > 7′·0	Total.
A	8	12	9	9	0	7	45
B	2	11	9	4	3	9	38
C	7	16	9	6	1	2	41
D	18	12	8	12	3	0	53
E	7	11	9	4	3	11	45
F	4	8	16	6	3	5	42
G	6	7	10	3	3	9	38
H	6	11	14	2	3	4	40
J	8	14	11	8	3	0	44
K	1	5	6	1	1	0	14
L	7	9	6	6	3	4	35
M	2	15	12	10	1	7	47
N	7	9	18	12	0	4	50
P	3	15	10	7	0	1	36
Q	7	12	10	9	5	5	48
R	4	5	9	3	3	2	26
S	2	10	10	9	1	5	37
T	5	6	14	11	2	7	45
	104	188	190	122	38	82	724

TABLE IV.

Distribution of the Poles of Groups U_n *and* U_s *with regard to Type.*

Position Letter.	h B.N.	Cp. Sym. Oval.	1 b.	2 b.	Cd.	py. abn. irr. py.	irr. unclassified.	φ	Total.
A	3	5	0	1	3	3	2	3	20
B	2	3	0	4	5	5	8	0	27
C	2	5	0	2	2	3	2	0	16
D	3	1	0	1	0	2	5	1	13
E	0	6	0	0	8	11	2	1	28
F	0	6	0	4	1	3	1	0	15
G	1	2	0	0	3	7	3	1	17
H	4	5	0	1	2	4	5	4	25
J	2	7	0	1	4	5	3	2	24
K	1	3	0	0	1	1	4	0	10
L	2	6	0	4	1	5	3	0	21
M	1	10	1	3	3	4	2	0	24
N	4	3	1	0	3	12	3	0	26
P	6	4	0	0	4	2	5	1	22
Q	1	3	1	1	2	6	4	3	21
R	3	2	0	2	1	2	0	2	12
S	2	10	1	3	3	3	5	0	27
T	3	5	2	1	0	6	3	0	20
	40	86	6	28	46	84	60	18	368

Addendum.

Since the above was written Mr. J. H. Reynolds very kindly informed the writer that two other papers had appeared on the subject. These must now be considered.

Mr. H. Knox-Shaw (see *M.N.*, 69, 498) found the positions of the poles of 25 large spirals, and his results, though tentative owing to the small number of nebulæ considered, are not in disagreement.

B. Meyermann (see *Ast. Nach.*, Band 219, Nr. 5240) has computed the right ascensions and declinations of the poles of 150 nebulæ from *Lick Observatory Publications*, 13. With the exception of 3 planetary nebulæ, 8 objects $< 1'$ in size and 6 larger spirals, all his objects are included in Table I. of this paper. There are a few numerical disagreements which I have investigated, but none of these affect the areas K and K′.

If Meyermann's figures be treated as above, we have the following table (in which a few numerical corrections have been made) to compare with Table II.

TABLE V.

Distribution of Poles.

Position Letter.	(1) $b \neq a$ $b \neq o$.	(2) $a = b$ $b = o$.	(3) (1)+(2) −4.
A	10	4	10
B	9	6	11
C	10	3	9
D	7	9	12
E	13	6	15
F	10	4	10
G	7	1	4
H	11	2	9
J	5	5	6
K	3	0	−1
L	9	4	9
M	10	4	10
N	10	3	9
P	5	8	9
Q	8	7	11
R	3	4	3
S	9	3	8
T	11	2	9
	150	75	153

As is to be expected, the results of both selections of nebulæ are very similar, at least as to the poles avoiding the areas K and K′.

Meyermann's selection gives $E_2(K)/E_2 = 3{\cdot}80$, and R and R′ again come second in paucity of poles. His figures, however, show a rather pronounced *concentration* of poles in E and E′, where $E_2(E)/E_2 = 2{\cdot}60$.